

Ch.1.1-1.4

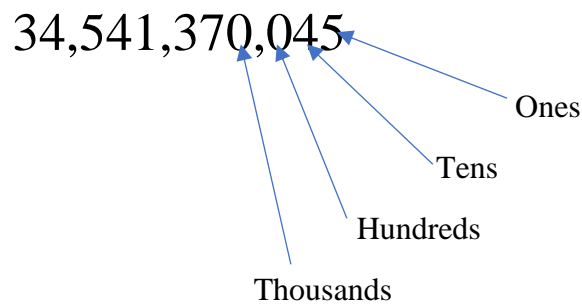
Reading whole numbers

You probably already know how numbers are grouped into *periods* using a comma. We start from the right side of a number and put a comma after every third *digit* like so:

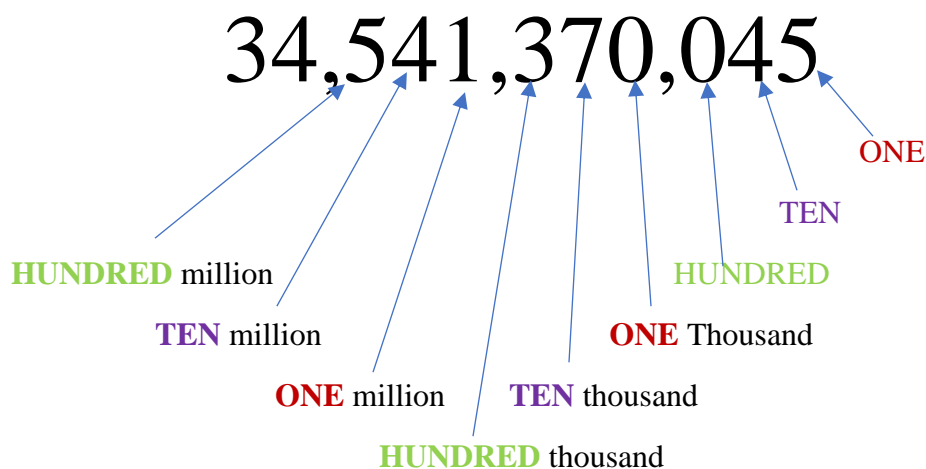


Each of the circles is around one period and you'll notice the last period only has 2 digits in it.

Every digit in a number has a *place value*. You may recall for example, that the “1” in the number above does not really mean “1,” but actually means 1 million. We know this because the way that place values are determined is pretty simple. The first place is the “ones” then “tens” then “hundreds” and so on, like this:



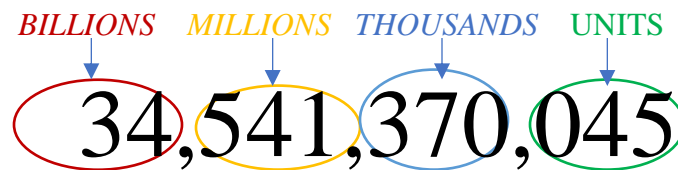
In truth, all you need to remember with place values is that *in every period, the first digit is “one” something, the second digit is “ten” something and the last digit is “hundred” something*. Only what comes after the “one,” “ten” or “hundred” changes. That may seem a little confusing, so let me show you what I mean here:



Thus, **ONE**, **TEN**, **HUNDRED**, **ONE** thousand, **TEN** thousand, **HUNDRED** thousand, **ONE** million, **TEN** million, **HUNDRED** million, **ONE** billion, **TEN** billion, **HUNDRED** billion and so on.

Write Whole Numbers

So, now that we know the place values, we can easily give a name to any number. It's easy because all we do is name each period with the digits that are in it from left to right. The first period (**units**) does not have a name because the numbers in it are used to name all the other periods. For example:



As you can see, there are 34 **BILLIONS**, 541 **MILLIONS**, 370 **THOUSANDS** and 45 **UNITS**. So, this number would be named:

*Thirty four **BILLION**, five hundred forty one **MILLION**, three hundred seventy **THOUSAND**, forty five.*

Notice that there is **NO “and”** anywhere in the name. It is NOT:

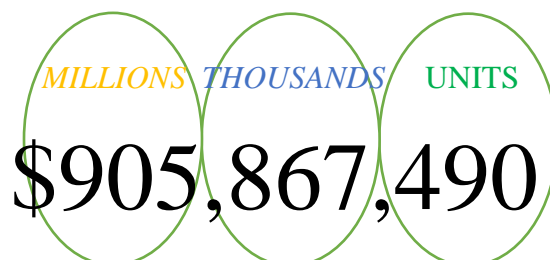
*Thirty four **BILLION**, five hundred forty one **MILLION**, three hundred seventy **THOUSAND** **AND** forty five, although people often say it this way.*

The word “and” will not appear in naming a number until we get to decimals.

Now, let's do it the other way around. In other words, take the words and make them into digits. Of course, all we have to do is think in reverse of what we just did. We take the digit 'value' in each period and put it in that period. For example, your boss tells you that last year, your company made:

*Nine hundred five **MILLION**, eight hundred sixty seven **THOUSAND**, four hundred ninety dollars*

Now you need to make a number out of that. Well, there are 905 **MILLIONS**, 867 **THOUSANDS**, 490 **UNITS**. So that means:



Looks like you deserve a bonus!

Round whole numbers

Now, practically speaking, when someone asks you how much your company made last year, are you going to tell them

Nine hundred five million, eight hundred sixty seven thousand, four hundred ninety dollars

every time? Of course not! You are actually **rounding**, which means you're giving a number that isn't exact, but close enough for the purposes at hand. In this case it just makes more practical sense to simply say:

Nine hundred million dollars

Many times, in real life we get into situations in which rounding a number makes sense, like this one. But to what place value we round it is very important. When you say:

Nine hundred million

you are rounding to the HUNDRED *millions* place. This means that you are only stating the value in the hundred *millions* place and everything after that is zero. Instead of saying this number:

\$905,867,490

You are saying this number

\$900,000,000

because it is more practical to say this. Obviously you would tell your accountant the exact amount, but for employees, \$900,000 is good enough.

Sometimes though, there are numbers in front of the place we are rounding to and we have to leave them too. For example, let's say I just drove:

453 miles

Now you probably already know just from life experience, that we are going to round this to:

450 miles

when we tell people how far we drove. Why? Because if we say 400, that's too small. If we say 500, that's just overdoing it. In any case, what we are doing here is rounding to the **TENS** place. Like before, everything AFTER the place value we are rounding to becomes a zero.

But how do we know to round up or down? Why didn't we say 460 miles above? Simply put, since we are rounding to the **TENS** place, we are essentially asking the question: is 453 closer to 450 or 460? The easiest way to do this is look at the number FOLLOWING the place we are rounding to. If it is a 5 or higher, we round up. If it is a 4 or lower, we keep it the same. In both cases, everything after the place value we are rounding to becomes a zero.

Here's an example:

19,283,746

Now let's round this number to each place value:

First, let's round to the **TENS** place:

TENS place

19,283,746

Because the "4" is in the **TENS** place and the number after it (6) is 5 or larger, we round up. Also, 19,283,746 is closer to 19,283,750 than 19,283,740, so our answer is:

19,283,750

If we round to the **HUNDREDS** place, we get:

ONE hundreds place

19,283,746

Because the "7" is in the **HUNDREDS** place and the number after it (4) is 4 or smaller, we keep it the same. Also, 19,283,746 is closer to 19,283,700 than 19,283,800, so our answer is:.

19,283,700

If we round to the **THOUSANDS** place, we get:

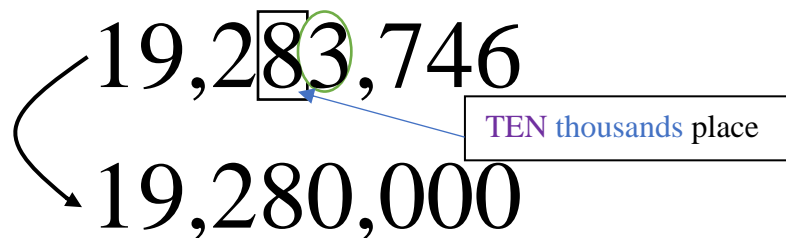
ONE thousands place

19,283,746

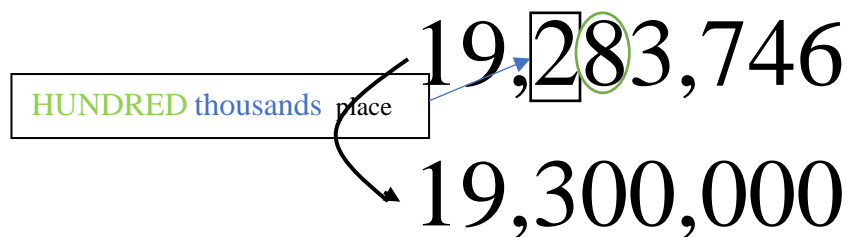
Because the “3” is in the **THOUSANDS** place and the number after it (7) is 5 or larger, we round up. Also, 19,283,746 is closer to 19,284,000 than 19,283,000, so our answer is:.

19,284,000

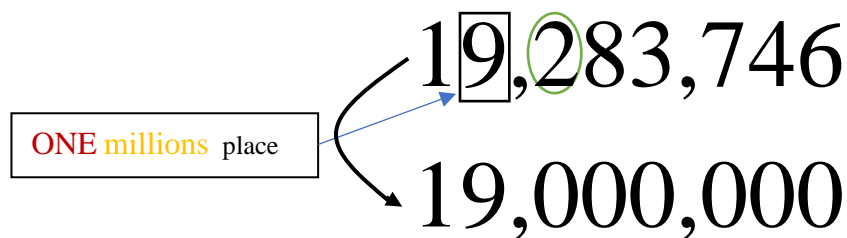
If we round to the **TEN thousands** place, we get:



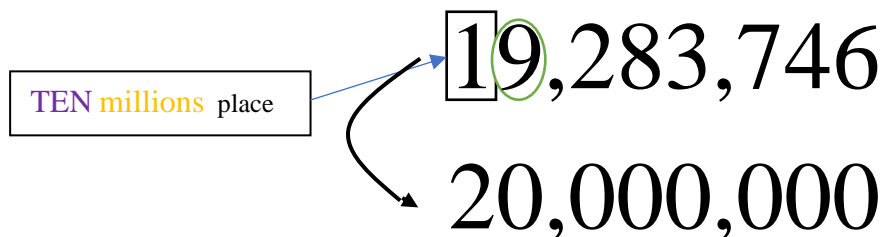
If we round to the **HUNDRED thousands** place, we get:



If we round to the **ONE millions** place, we get:



If we round to the **TEN millions** place, we get:



Read and Round Integers

Sometimes numbers can also be negative. In business for instance, you might have a **loss**, which is another way of saying that your profit is negative. Let's take the example we used before of your company making:

\$905,867,490

Let's say instead that this represents a loss, not a profit. In other words, the company spent \$905,867,490 more than they made. Ouch, I guess no bonus for you this time!

In any case, the **ONLY** difference when dealing with this number in terms of writing it and naming it is including a negative sign or the word "negative."

Thus, we would write this number as:

-\$905,867,490

And we would say:

***NEGATIVE** Nine hundred five **MILLION**, eight hundred sixty seven **THOUSANDS**, four hundred ninety dollars*

Add and subtract whole numbers

Numbers that are being added together are called **addends** and the result is the **sum** or **total**. For example:

$$\begin{array}{c} \textcircled{3 + 5} = \textcircled{8} \\ \uparrow \qquad \qquad \uparrow \\ \text{Addends} \qquad \text{Sum} \end{array}$$

There are some properties of addition that I bet you already kind of know, but maybe didn't realize.

- **Commutative Property of Addition:** This simply means that we can add in any order we want. Take the sum above, we could also write:

$$5 + 3 = 8$$

- **Associative Property of Addition:** This property isn't much different than the last one but deals with more than two numbers being added together. For example:

$$(3 + 4) + 5 = 12$$

$$3 + (4 + 5) = 12$$

In both of these, the parentheses tell us what to do first. In #1 we add $3 + 4$ first and then add that to 5. In the second, we add $4 + 5$ first and then add 3. In both cases we get twelve.

So, both the commutative and associative properties of addition essentially tell us that we can add in any order we want and we'll get the same answer.

But keep in mind, this only applies when we are *just* adding. Neither of these properties applies to subtraction. Here's what I mean

$$5 - 3 = 2$$

$$3 - 5 = -2$$

Minuend Subtrahend Difference

This is why there is no commutative or associative property of subtraction, but you may see that there is a pattern here.

Likewise, if we have a problem with multiple numbers and subtraction, what we do first *does* make a difference. For example:

$$(7 - 5) - 2 = 0$$

$$7 - (5 - 2) = 4$$

In the first problem we did $7 - 5$ first and then subtracted 2. In the second, we did $5 - 2$ first and then subtracted that from 7. But we got a different answer, unlike when we did the same thing with addition. Thus, we can add in any order we want, but we *cannot* subtract in any order we want.

Estimation

Whenever you do are adding or subtracting, it is a good idea to estimate before you get the exact answer to make sure you're on the right track. For example, take this:

645 + 123

This is about $600 + 100$, which is 700. Of course, we know our answer will be a bit more than that but at least in that ballpark. Let's say we add it by hand or calculator and make a mistake and get 7068. We should automatically realize we did something wrong because it is nowhere near what it should be. In fact, the answer is 768.

Add and subtract integers

In business we deal with positive and negative numbers all the time. Positive numbers are usually used to indicate an increase of some kind like revenue or profit. Negative numbers indicate a decrease such as a loss or money that is owed. Thus, it is essential to understand how we put these numbers together.

Add two negative numbers

Let's say that your business had a loss of \$5,000 last year and \$6,000 the year before. We could say that the total would be:

| | | |
|-----------------------------------|-----------------|---------------------------|
| Loss for year 1 | Loss for year 2 | Total loss for both years |
| ↓ | ↓ | ↓ |
| $-\$5,000 + -\$6,000 = -\$11,000$ | | |

You may notice that adding two negatives is the same as adding two positives, but with a negative sign. Adding two negative numbers will always result in a negative number.

Add a positive and negative number

It is important to understand that ***adding a negative number is the same as subtracting a positive number***. For example, let's say that you made \$35,000 in revenue last month with a cost of \$12,000 and you want to figure out how much profit you made. The first way you could think of this is probably the most intuitive; revenue minus costs, like this:

| | |
|-----------------------|-------|
| revenue | costs |
| ↓ | ↓ |
| $\$35,000 - \$12,000$ | |


But you could also think of it as your revenue plus your costs, which are negative, like this:

| | |
|--------------------------|-------|
| revenue | costs |
| ↓ | ↓ |
| $\$35,000 + (-\$12,000)$ | |

Both of these give us the same answer (\$23,000) because mathematically, they are the same.

Change a subtraction problem to addition

We could even think of it as our costs (which are negative) plus our revenue because, remember, we can add in any order we want and this includes adding negative numbers too:


$$\begin{array}{c} \boxed{\text{revenue}} \qquad \qquad \boxed{\text{costs}} \\ \downarrow \qquad \qquad \qquad \downarrow \\ -\$12,000 + \$35,000 \end{array}$$

Dealing with subtracting a negative

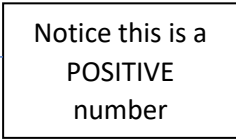
This one is a little harder to understand, so I will use an example. Let's say it's 20° outside in the afternoon and by nightfall it is 5°. How much did the temperature change? You probably had no trouble figuring out that the change is 15° because you did:

$$20^{\circ} - 5^{\circ} = 15^{\circ}$$

What we really did here is take the starting temperature and subtract the final temperature. That gives us a change of 15°. But what if it started out at 20° and ended up at -5°? You probably already realize that this is a difference of 25° but why? From 20 to zero and zero to -5 is a total of 25. But think about the math we just did. Remember before, we took the starting temperature minus the ending temperature, which is *always* how we determine a change in something (in this case temperature). If we do that here, it gives us this:

$$20^{\circ} - (-5^{\circ})$$

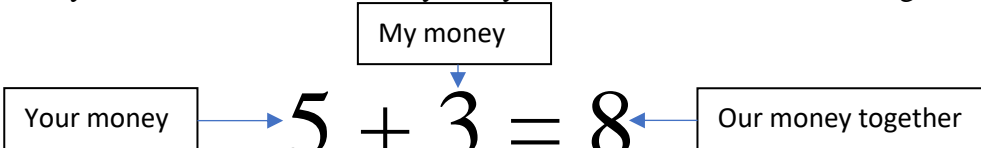
But we already know that answer is going to be *positive* 25°, don't we? That has to mean that:

$$20^{\circ} - (-5^{\circ}) = 25^{\circ}$$


Notice this is a
POSITIVE
number

But if $20 - (-5) = 25$ and $20 + 5 = 25$, these two problems must be the same and this is why *subtracting a negative is the same as simply adding*.

Here's another way to think about it. Let's say that you have \$5 and I have \$3. Together, we have:


$$\begin{array}{c} \boxed{\text{My money}} \\ \downarrow \\ \boxed{\text{Your money}} \rightarrow 5 + 3 = 8 \leftarrow \boxed{\text{Our money together}} \end{array}$$

The difference between our cash is:

$$\begin{array}{ccccc} & & \text{My money} & & \\ & & \downarrow & & \\ \text{Your money} & \rightarrow & 5 - 3 = 2 & \leftarrow & \text{The difference between our money} \end{array}$$

Notice that we did your cash minus my cash because your cash is bigger than mine. If we did it the other way around, we'd get a negative number and that wouldn't make as much sense. Now, what if these were debts? You owe \$5 and I owe \$3. Together, we would have:

$$\begin{array}{ccccc} & & \text{My debt} & & \\ & & \downarrow & & \\ \text{Your debt} & \rightarrow & -5 + -3 = -8 & \leftarrow & \text{Our "money" together} \end{array}$$

To find the difference between our cash (or lack thereof) would still be the same operation we did before. We just subtract two numbers. But remember, before we did your cash minus mine because you had more. Since we're dealing with debts now, I have more 'money' than you do so we do mine minus yours. But I have no doubt you already know the answer is 2. So, let's look at the math:

$$\begin{array}{ccc} & \text{My debt} & \\ & \downarrow & \\ \text{Your debt} & \rightarrow & -3 - (-5) \end{array}$$

Like I said, the answer has to be 2 (that's the difference between -5 and -3, isn't it?), so:

$$\begin{array}{ccccc} & \text{Your debt} & & & \\ & \downarrow & & & \\ \text{My debt} & \rightarrow & -3 - (-5) = 2 & \leftarrow & \text{The difference between our debt} \end{array}$$

But that gives us the same answer as:

$$-3 + 5$$

In other words, these are the same! And again, this is why subtracting a negative is the same as simply adding a positive.

Here are a few more examples to help you see:

$$3 - (-5) = 3 + 5$$

$$7 - (-9) = 7 + 9$$

$$-14 - (-5) = -14 + 5$$

$$-3 - (-5) = -3 + 5$$

$$10 - (-5) = 10 + 5$$

Properties of Multiplication

There are some properties of multiplication that are pretty much the same as those for addition. In fact, we'll use the same numbers we did with addition:

- ***Commutative Property of Multiplication***: This simply means that we can multiply in any order we want. For example:

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

- ***Associative Property of Multiplication***: This property isn't much different than the last one but deals with more than two numbers being multiplied together. For example:

$$(3 \times 4) \times 5 = 60$$

$$3 \times (4 \times 5) = 60$$

In both of these, the parentheses tell us what to do first. In #1 we multiply 3×4 first and then multiply that by 5. In the second, we multiply 4×5 first and then multiply 3. In both cases we get 60.

So, both the commutative and associative properties of multiplication essentially tell us that we can multiply in any order we want and we'll get the same answer.

But keep in mind, this only applies when we are *just* multiplying. Neither of these properties applies to division. Here's what I mean

$$5 \div 3 = 1.7$$

$$3 \div 5 = 0.6$$

This is why there is no commutative or associative property of division.

Likewise, if we have a problem with multiple numbers and division, what we do first *does* make a difference. For example:

$$(7 \div 5) \div 2 = 0.7$$

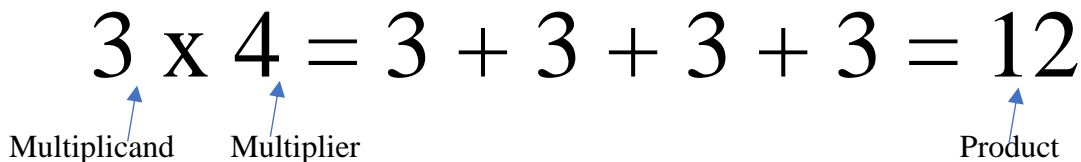
$$7 \div (5 \div 2) = 2.8$$

In the first problem we did $7 \div 5$ first and then divided by 2. In the second, we did $5 \div 2$ first and then divided that by 7. But we got a different answer, unlike when we did the same thing with multiplication. Thus, we can multiply in any order we want, but we *cannot* divide in any order we want.

Let's multiply!

Multiplication is really a shortcut to addition. Think about it. You probably already know that 3×4 means four 3's, like this:

$$3 \times 4 = 3 + 3 + 3 + 3 = 12$$



Multiplicand Multiplier Product

When we are multiplying multiple digit numbers together, we use a logarithm, a fancy name for a process, to do this. You probably remember it from as long ago as grade school, but let's talk about what it means. Let's say that we have this problem:

$$\begin{array}{r} 34 \\ \times 7 \\ \hline \end{array}$$

Now, you'll likely recall that first we multiply the 7 by the 4 and get 28. We write the 8 under the seven and "carry" the 2 like this:

$$\begin{array}{r} 234 \\ \times 7 \\ \hline 8 \end{array}$$

Next, we multiply the 7 by the 3, giving us 21, but we also add the 2 that we carried, giving us 23. We write the 3 and carry the 2, giving us this:

$$\begin{array}{r} 234 \\ \times 7 \\ \hline 38 \end{array}$$

But you'll notice that there is no hundreds place, which is the same as being zero. $7 \times 0 = 0$, plus the two that we carried, gives us 2 and we get our final answer:

$$\begin{array}{r} 34 \\ \times 7 \\ \hline 238 \end{array}$$

Most of us know how to do this but often don't quite realize why. Notice that the "4" and "7" are both in the ones place, so when we do the 7×4 part of the problem, we get 28. But also, because the "4" and "7" are both in the ones place, we "carry" the "2" to the tens place. It is actually two tens. But the "3" is in the tens place, which means that when we do the 7×3 part of the problem, we're really doing 7×30 , which gives us 210. In other words, we can break down

this problem into parts that hopefully gives you some insight into WHY we do multiplication the way we do.

$$\begin{array}{r} 30 \\ \times 7 \\ \hline 210 \end{array} + \begin{array}{r} 4 \\ \times 7 \\ \hline 28 \end{array}$$

When we multiply numbers with at least two digits each, the process is the same but we have to extend it. Say we now have:

$$\begin{array}{r} 34 \\ \times 57 \\ \hline \end{array}$$

Just like before, we do $7 \times 4 = 28$. We write the 8 and carry the 2 like so:



$$\begin{array}{r} 234 \\ \times 57 \\ \hline 8 \end{array}$$

Again, as before, we do $7 \times 3 = 21$, plus the “2” we carried gives us 23:

$$\begin{array}{r} 34 \\ \times 57 \\ \hline 238 \end{array}$$

You’ll notice that this is the same as the problem before. We multiplied the 7 by the 4 and by the 3 (which is really 30). Now we still have to multiply the “5” by everything. But notice the 5 is in the tens place and is thus, really “50.” This means that when we multiply the “5” by the “4,” we have to put the answer in the tens place, which is here:

$$\begin{array}{r}
 34 \\
 \times 57 \\
 \hline
 238
 \end{array}$$

tens place   ones place

Since $5 \times 4 = 20$, we write the “0” and carry the “2” like this:

$$\begin{array}{r}
 ^2 34 \\
 \times 57 \\
 \hline
 238 \\
 0
 \end{array}$$

Now we do $5 \times 3 = 15$, plus the “2” that we carried, gives us 17. Now we have:

$$\begin{array}{r}
 34 \\
 \times 57 \\
 \hline
 238 \\
 170
 \end{array}$$

Now we add these products together to get the final answer:

$$\begin{array}{r}
 34 \\
 \times 57 \\
 \hline
 238 \\
 +170 \\
 \hline
 1938
 \end{array}$$

To see what we really did, it might help to see it this way:

$$\begin{array}{r}
 34 \qquad 34 \\
 \times 50 \quad + \quad \times 7 \\
 \hline
 1700 \quad + \quad 238
 \end{array}$$

You simply follow the same pattern to multiplication of two numbers of any size. Here's one that I already worked out so that you can see the pattern. You will not be required to do one this big, but I simply want you to see that the pattern is the same, even for huge numbers:

Expanded form

When we think of

$$\begin{array}{r}
 34 \\
 \times 57
 \end{array}
 \text{ as the same as }
 \begin{array}{r}
 34 \qquad 34 \\
 \times 50 \quad + \quad \times 7 \\
 \hline
 \hline
 \end{array}$$

we are taking advantage of what is called ***expanded form***. This is when we break a number into parts based on place value. Sometimes this can be useful, such as with this problem. The best way to explain this is with an example:

$$12,345 = 10,000 + 2,000 + 300 + 40 + 5$$

The number 12,345 in expanded form

Zero shortcut

Sometimes really big numbers can easily be multiplied together, even in your head, if they have a bunch of zeros after them. For instance, let's say your business has 1800 gas stations across the country and on average, each sells 100,000 gallons of gas each year. How much gas is that?

Well, we could set it up like above, and it would work, but there is an easier way. I'm not going to try to show you exactly why because it might only confuse you, but because of the way our number system works, we can apply the zeros at the end of a multiplication problem whenever we want to. What we have is this:

$$\begin{array}{r} 100000 \\ \times 1800 \\ \hline \end{array}$$

But instead, I can remove all the zeros, and remember you can only do this with the zeros at the *end* of a number, do the math and then add them back on. So, I have this:

$$\begin{array}{r} 1 \\ \times 18 \\ \hline \end{array}$$

(7 zeros waiting patiently)

000000
00

Now all we have to do is 1 x 18 and then add all the zeros back on like this:

$$\begin{array}{r} 1 \\ \times 18 \\ \hline 18 \end{array}$$

000000
00

7 zeros return

$$= 180000000 \leftarrow 7 \text{ zeros}$$

Multiplying positive and negative numbers

You may already be familiar with the fact that when we multiply two positives, we get a positive. Two negatives and we get a negative. One of each and we get a negative. The reason is because of what multiplication means as explained before. Remember:

$$3 \times 4 = 3 + 3 + 3 + 3$$

But what if one of them is negative? Now instead of four 3's, we have four (-3)'s like this:

$$-3 \times 4 = -3 + -3 + -3 + -3$$

We already know how to do this and of course the answer is -12. Take note that we multiplied a negative and positive number together and it was a negative answer. When we multiply two negative numbers together, we get positive. It is sort of like one negative undoing the other. You might say two 'wrongs' make a 'right' in this case. I will not attempt to prove to you why this is so because again, I think it would confuse you at this point. So, remember that when multiplying two numbers together, the answer will be positive if the signs are the same (two negatives or two positives) and negative if they are different.

When we are multiplying several numbers together, we simply multiply two at a time until we are done, using what we already know. For example, let's say we have this:

$$2 \times (-4) \times (-7) \times 9$$

As with addition, we can multiply in any order we want, but let's just go from left to right.

$$2 \times (-4) = -8$$

$$-8 \times -7 = 56$$

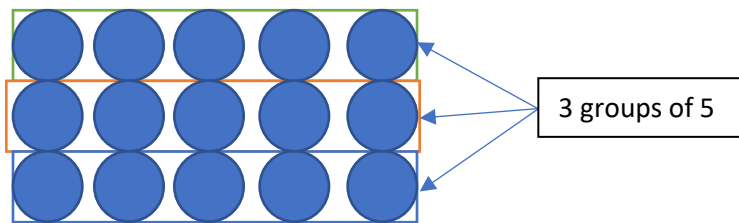
$$56 \times 9 = 504$$

Dividing

Division simply means that we are seeing how many times a number can ‘go in’ to another number. For instance:

$$15 \div 3$$

What this means in words is “15 divided by 3” or “how many times does 3 go into 15?” We can also look at this from a visual point of view by taking 15 circles and dividing them into 3 equal-sized groups:



And of course, the answer is 5. You’ll also notice that:

$$15 \div 3 = 5$$

dividend

quotient

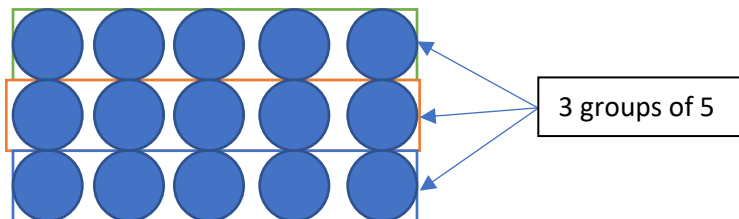
divisor

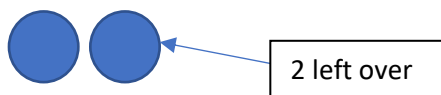
And

$$5 \times 3 = 15$$

This is because division is just multiplication in reverse, just like subtraction is addition in reverse. But sometimes the *divisor* doesn’t get divided by the *dividend* an even number of times and we have some ‘leftovers,’ called the *remainder*. You can see this when we do this problem:

$$17 \div 3$$





Thus:

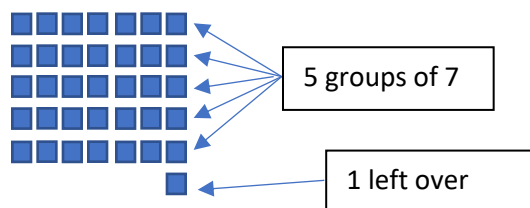
$$17 \div 3 = 5 \text{ R}2$$

means remainder

So, let's say that you are a District Manager for GNC stores. There are 36 stores in your district and you need to contact each store at least once a week. Since you are the manager, you don't work weekends, so there are 5 days each week that you can contact stores. How many do you need to contact each day to make sure you get them all. Well, let's take the *number of stores* and divide them by the number of *days per week*. That will give us *stores per day*.

$$36 \div 5 = 7 \text{ R}1$$

Or visually



So, you will need to call 7 stores a day and since you have one left over, you'll need to call an 8th store on one of those days.

Dividing bigger numbers

So far, we have only been dividing numbers that we can more or less do in our head. But what about bigger ones that can't, like:

$$487 \div 3$$

You may recall as far back as grade school that we could set this up like so:

$$3 \overline{) 487}$$

Just like multiplication, we use an algorithm to solve this and what we essentially do is divide 3 into a bunch of different smaller pieces that we *do* know the answer to and then combine them all. We'll go through the actual process in a minute, but the reason it works is because we are basically dividing 3 into 400, 3 into 80 and 3 into 7 since the "4" is in the ONE hundreds place, the "8" is in the TENS place and the "7" is in the ONES place like this:

$$\begin{array}{r}
 400 \div 3 \\
 80 \div 3 \\
 + \quad 7 \div 3 \\
 \hline
 \end{array}$$

So, let's go back to the original equation:

$$3 \overline{)487}$$

First, we have to determine if the "3" will go into the "4" and it will, one time. But when we take one 3 from the 4, we have one left over. We still have not divided the "3" into the leftover "1" yet, so we carry it along like so:

$$\begin{array}{r}
 1 \\
 3 \overline{)487} \\
 \underline{-3} \\
 1
 \end{array}$$

Now, we still haven't divided the "3" into the "1," but 3 doesn't go into one a whole time. So, we bring down the 8 because we haven't divided into it yet either:

$$\begin{array}{r}
 1 \\
 3 \overline{) 487} \\
 \underline{-3} \\
 18
 \end{array}$$

3 does go into 18, 6 times and nothing left over. That gives us this:

$$\begin{array}{r}
 16 \\
 3 \overline{) 487} \\
 \underline{-3} \\
 18 \\
 \underline{-18} \\
 0
 \end{array}$$

Now we still haven't divided into the "7" yet, so we bring it down.

$$\begin{array}{r}
 16 \\
 3 \overline{) 487} \\
 \underline{-3} \\
 18 \\
 \underline{-18} \\
 07
 \end{array}$$

3 will go into 7 two times with a remainder of 1. So now we have this:

$$\begin{array}{r}
 162 \\
 3 \overline{) 487} \\
 \underline{-3} \\
 18 \\
 \underline{-18} \\
 07 \\
 \underline{-6} \\
 1
 \end{array}$$

Since there is nothing left to divide into, our answer is:

162 R1

Dividing with more than 1 digit in the divisor

Sometimes we will divide a number by another number bigger than 10. But don't worry, the process is EXACTLY the same way. Let's walk through an example similar to the one we just did and you'll see what I mean:

First, we have to determine if the "13" will go into the "4" and of course it doesn't, so we put "0". However, "13" does go into 48 three times. But when we take three 13's from the 48, we have 9 left over. We still have not divided the "13" into the leftover "9" yet, so we carry it along like so:

$$\begin{array}{r} 03 \\ 13 \overline{) 487} \\ - 39 \\ \hline 9 \end{array}$$

Now, we still haven't divided the "13" into the leftover "9," but 13 doesn't go into 9 a whole time. So, we bring down the 8 because we haven't divided into it yet:

$$\begin{array}{r} 03 \\ 13 \overline{) 487} \\ - 39 \\ \hline 97 \end{array}$$

13 does go into 98, seven times with 6 left over. So now we have this:

$$\begin{array}{r}
 037 \\
 13 \overline{) 487} \\
 \underline{- 39} \\
 97 \\
 \underline{- 91} \\
 6
 \end{array}$$

Since there is nothing left to divide into, our answer is:

$$37 \text{ R}6$$

Dividing positive and negative numbers

Remember how I said that division is just backwards addition? Actually, you can see this in the last problem. We did:

$$487 \div 3$$


And got the answer:

$$162 \text{ R}1$$

If we do it in reverse, we get:

$$\begin{array}{r}
 162 \\
 \times 3 \\
 \hline
 486
 \end{array}
 + 1 = 487$$

remainder



But what about when negative numbers creep into division? Well, you'll be happy to know that since division *is* multiplication in reverse, the relationship works the same. Remember, if the

signs were the same in multiplication (2-negatives or 2-positives), the answer is always positive? The same thing applies when doing division and for the same reason. So:

$$487 \div 3 = 162 \text{ R}1$$

$$-487 \div 3 = -162 \text{ R}1$$

$$487 \div -3 = -162 \text{ R}1$$

$$-487 \div -3 = 162 \text{ R}1$$

Another zero shortcut

You may be getting sick of me saying how division is just backwards multiplication, but once you *really* recognize this fact, it will no longer be a process to do, but you will understand it like you understand how to walk. That being said, just like the zero shortcut we have with multiplication (taking off the zeros, multiplying and then adding them back on to the answer), there is one in division too. Think about this for a second:

$$6 \div 3 = 2$$

Or

$$\begin{array}{r} 2 \\ 3 \overline{) 6} \end{array}$$

3 goes into 6 two times, right? Now what about this:

$$60 \div 30 = 2$$

Or

$$\begin{array}{r} 2 \\ 30 \overline{) 60} \end{array}$$

30 also goes into 60 two times, doesn't it? How about:

$$600 \div 300 = 2$$

Or

$$\begin{array}{r} 2 \\ 300 \overline{) 600} \end{array}$$

I hope you see the pattern here between the zeros and the multiplication. You can remove *the same number of zeros* on the divisor and the dividend and you have the same equation, but it might be much easier to do. Here's an example:

$$600 \div 12,000 = 20$$

Or

$$600 \overline{) 12000}$$

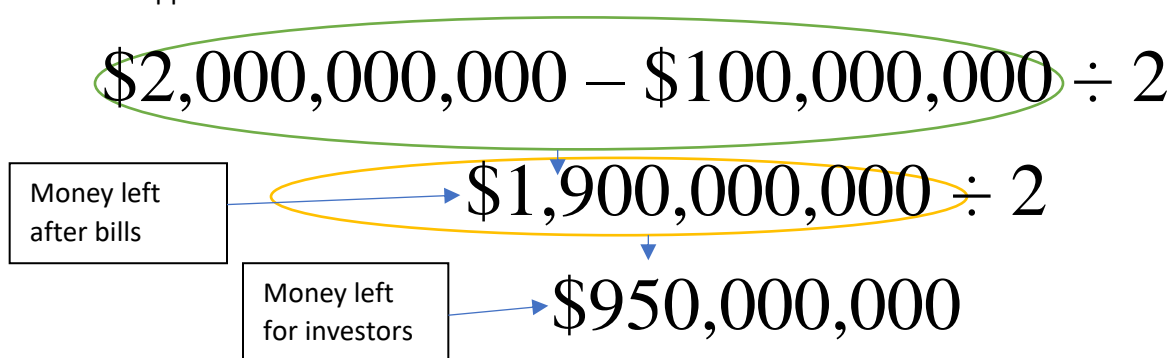
If we take 2 zeros from both numbers, we have exactly the same division going on, but it is much easier to do. We have this:

$$6 \overline{) 120}$$

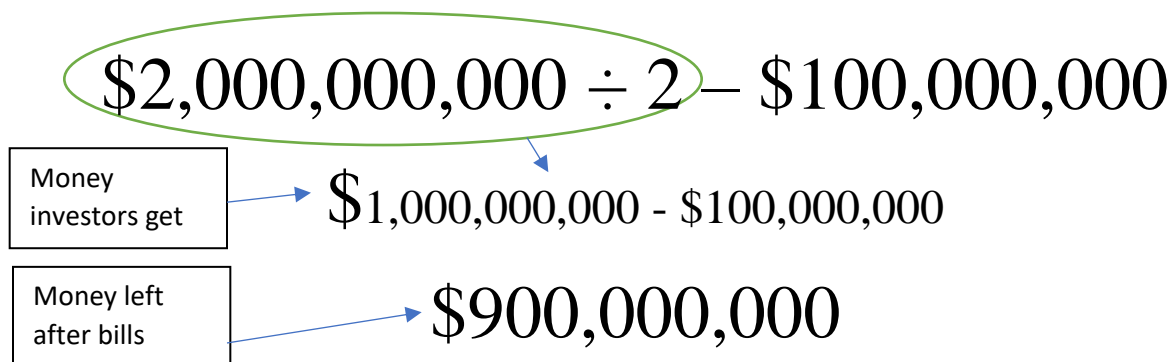
Order of Operations

Order of operations simply refers to the order in which we do math when we have a bunch of numbers with different operations. We now know how to deal with numbers that are all being added or multiplied, for example, but what if there is addition and multiplication together? Or for that matter, division and subtraction.

But why does this even matter? Glad you asked! Let's pretend you are working as COO of Microsoft and the company made \$2 billion this quarter. Your job is to pay investors with half of that money and pay bills of \$100 million. In other words, we are going to be dividing and subtracting. So, does it matter what we do first? Pay the investors first (divide) or the bills first (subtract)? Let's pay the bills first and see what happens:



So, there is \$950,000,000 to spread among the investors. Now let's pay the investors first and see what the difference is.



So, if you pay the bills first, the investors only get \$950 million. If you pay the investors first, they get \$1 billion. That's a difference of 50 million bucks! In a word, "yes," order of operations does matter. So, let's look at another problem with much smaller numbers.

$$1 + 2 \times 5$$

Depending on what order we do this in, we get different answers. For example, let's just go from left to right and do what it says to do. We get this.

$$\begin{array}{c} 1 + 2 \times 5 \\ \downarrow \\ 3 \times 5 \\ \downarrow \\ 15 \end{array}$$

Now, for the sake of this problem, I will tell you that multiplication comes before addition, but more on that in a minute. So, let's do those first and see what happens:

$$\begin{array}{c} 1 + 2 \times 5 \\ \downarrow \\ 1 + 10 \\ \downarrow \\ 11 \end{array}$$

Interesting, we got different answers even though we did the same math as before, but in a different order.

The general rule we use to tell us what order to do operations is PEMDAS. Some people like to remember:

Please Excuse My Dear Aunt Sally

It tells us the order we are to do operations when we have a bunch of numbers. It means this:

Parenthesis Exponents Multiplication Division Addition Subtraction

In other words, the first thing we do is parenthesis, then exponents, then multiplication and division, then addition and subtraction. It is important to understand here that multiplication and division are equally important, but still come before addition and subtraction, which are also equally important. We can rewrite PEMDAS like this to emphasize this fact:

$$PE \frac{M}{D} \frac{A}{S}$$

So, let's solve a problem using PEMDAS. Let's say you have \$300. You buy a movie ticket for \$20, then find \$1 on the ground, then bet it all on horse and you make 6 times your money. After that, you divide it evenly among your 3 bank accounts. If we write all the math together that happened here, we get this:

$$(300 - 20 + 1) \times 6 \div 3$$

According to PEMDAS, the first thing we do is parenthesis, so we do what is inside of there first. You'll notice though, that there is addition and subtraction and I just said that they are equally important, so what do we do first? In this case it actually doesn't matter, but as a rule, you always do what is farthest to the left when you have two operations that are equally important. So, let's do $300 - 2$ first. That will give us this:

$$(280 + 1) \times 6 \div 3$$

Now add "1":

$$(281) \times 6 \div 3$$

At this point, there is nothing more to be done in the parenthesis, so they don't matter anymore and we have this:

$$281 \times 6 \div 3$$

PEMDAS tells us that exponents are next but we don't have any, so now we can move on multiplication and division. Now remember, I said they are equally important, so again, what do we do first? We will get very different answers depending on what we decide to do. Like we did with the addition and subtraction just a few seconds ago, we'll move from left to right. In other words, we'll do the multiplication first because it comes before the division when we read the problem (and we read left to right of course). Here's what we've got now:

$$1686 \div 3$$

This part is easy because there is only one thing left to do:

$$562$$

Here's one more:

$$3 + 2 \times (13 - 8 \div 4)$$

First we will do what is inside the parenthesis. Now, inside the parenthesis there is subtraction and division to be done. Of course, division always comes before subtraction, even though the subtraction is farther to the left. So now we have:

$$3 + 2 \times (13 - 2)$$

We're still not done with the parenthesis though:

$$3 + 2 \times (11)$$

Now that we have done all the math in the parenthesis, they aren't necessary anymore and we have:

$$3 + 2 \times 11$$

This time we have addition and multiplication to deal with. According to PEMDAS, multiplication comes first, which means we now have:

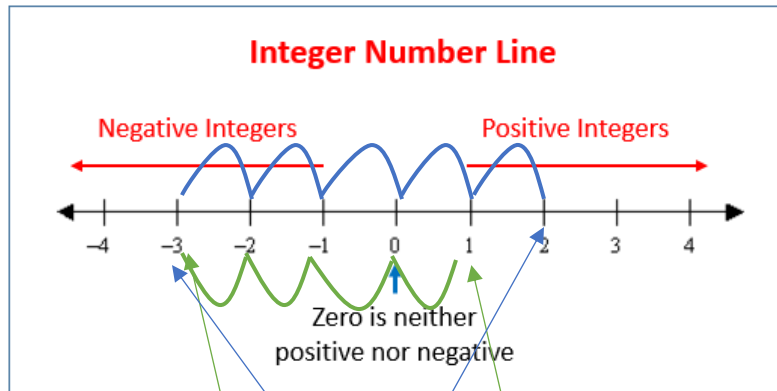
$$3 + 22$$

There's only addition left now and we're done:

$$25$$

Number lines

Sometimes representing a number on a number line can be helpful. It may not seem so much now but it will make even more sense in later math. For now, a number line is simply a line on which we mark a specific value. It allows us to visualize that number in comparison to other numbers. For example, here is one:



You'll notice that as we move right on the number line, the numbers become more positive because we are adding. For example, if we go from -3 to 1, we have moved 4 spaces in the positive direction. By the way, did you notice that:

$$-3 + 4 = 1$$

Of course, it works the other way too. When we move left, we are subtracting. For example, if we begin at 2 and move to -3, we have gone 5 spaces to the left. And,

$$2 - 5 = -3$$

Perimeter

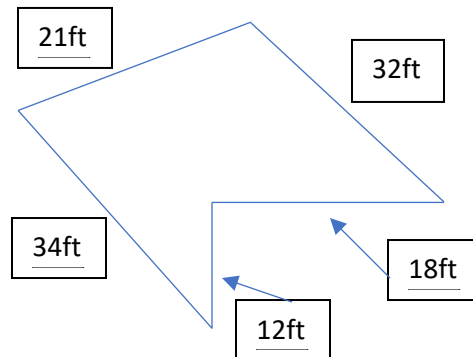
Now that we know all this math, we can use it to solve a real problem. Let's say that you need to build a fence around your yard and it is shaped like this:



The *perimeter* of *any shape* is just the distance around it. In this case, we have a rectangle and so two sides of it are 50ft and the other two sides are 60ft. Thus, we would simply add them all up to get the distance around this thing, which of course tells us how much fencing we need.

$$50\text{ft} + 60\text{ft} + 50\text{ft} + 60\text{ft} = 220\text{ft}$$

Even a weird shape would be the exact same thing. Perimeter is just the distance around, so let's say your yard looked like this instead:

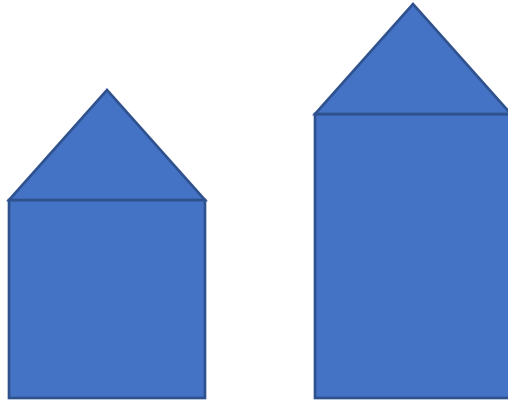


The perimeter of this would be done exactly the same way. It's just the distance around. So, we'd get:

$$\begin{array}{r} 34\text{ft} \\ 12\text{ft} \\ 18\text{ft} \\ 32\text{ft} \\ + 21\text{ft} \\ \hline 117\text{ft} \end{array}$$

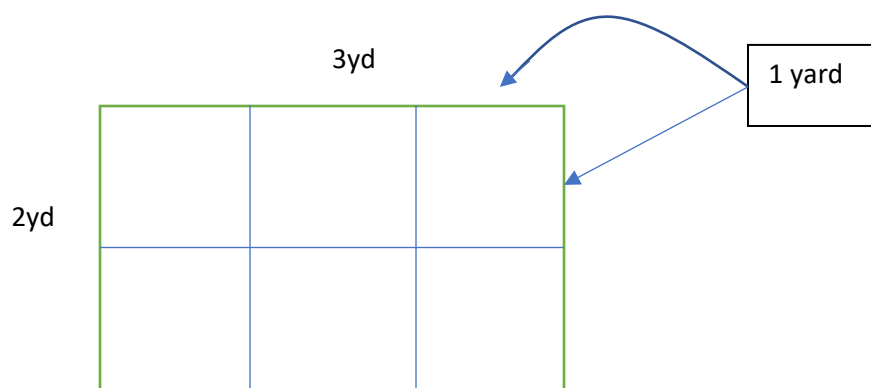
Area

Another important concept is area. This is just the amount of 2 dimensional space in a shape. in other words, a flat surface. Think about the area of a home for a second. Let's say you live in a 2000 square foot home. You have a sense of how big that is but what does it mean? One way to think of it is that if we had 2000 1ft x 1ft squares, they would exactly fit on the floor. We might have to cut some of them to fit each individual room but every scrap would ultimately have a place. So, if you have two one story homes that are exactly the same except one is taller than the other, do they have the same area or square footage inside?:



If we really think about it, the square feet in a home is really just the amount of floor space there is. So, two otherwise identical homes could both have 2000 square feet but with one taller than the other. Of course, the taller one has more volume, or 3-dimensional space inside it, but that isn't what we are talking about. We're talking about walking space. The same is the case with area. We are just talking about the "area" you could walk on.

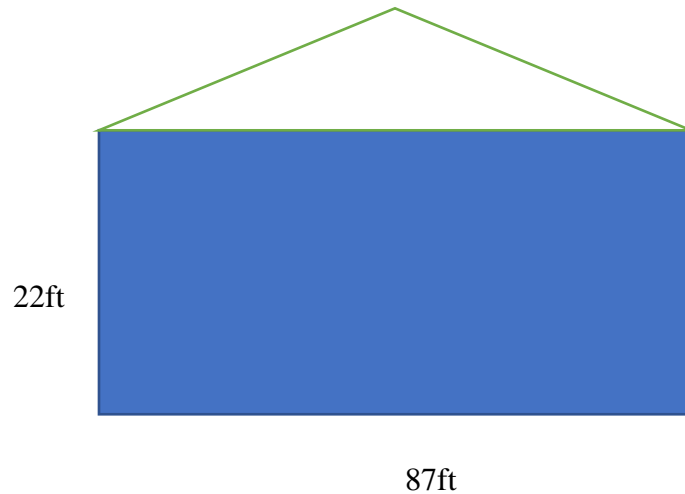
For now, we're just going to talk about rectangles. So, to help in explaining why the area is what it is, let's use a small, simple one.



You'll notice that I divided this into 1yd lengths, 3 on the long side of the rectangle and 2 on the short side. And not surprisingly, we get exactly 6 boxes that are 1yd by 1yd. Or in other words, 6 square yards. Another way to write that would be 6yd^2 . You'll likely also notice that $3 \times 2 = 6$. Coincidence? Of course not. In fact, the formula for the area of a rectangle is simply the length x width:

$$A = L \times W$$

Let's do one more. Pretend that you are going to paint one side of your barn, so of course you need to know the area that you will be painting in order to know how much paint to buy. Here's your barn:



Don't be confused with the roof, we're not painting that. Just the rectangular side of the barn. And since its length is 87ft and its width is 22ft, we get:

$$87\text{ft} \times 22\text{ft} = 1,914\text{ft}^2$$

Another way to interpret this is that if we had 1,914 1ft by 1ft squares, they would exactly cover the side of the barn.

Exponents

An exponent is a "power" and is a way of compressing multiplication in the same way that multiplication compresses addition. Remember:

$$3 \times 4 = 3 + 3 + 3 + 3$$

It's makes more sense to write this, than this

You can see how this is especially true when we have big numbers like this:

$$56 + 56 + 56 + 56 + 56 + 56 + 56 + 56 + 56 + 56 + 56 + 56$$

It would make a lot more sense to write this instead:

$$56 \times 12$$

Likewise, exponents do the same for multiplication. So instead of writing:

$$3 \times 3 \times 3 \times 3$$

We can write:

$$3^4$$

They mean exactly the same thing. Again, imagine how much more sense it makes to use an exponent, especially with big numbers like we did with multiplication. Imagine we have this:

$$56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56 \times 56$$

Instead, we can write:

$$56^{12}$$

The idea of exponents is even easier to understand when we have powers of 10. Here's a simple chart of a few for you to see the pattern:

| POWER | WHAT IT MEANS | EXPANDED VALUE |
|--------|------------------------|----------------|
| 10^1 | 10 | 10 |
| 10^2 | 10 x 10 | 100 |
| 10^3 | 10 x 10 x 10 | 1,000 |
| 10^4 | 10 x 10 x 10 x 10 | 10,000 |
| 10^5 | 10 x 10 x 10 x 10 x 10 | 100,000 |

Notice that the exponent corresponds to the number of zeros on the actual number. So now we could even do math with these guys. For instance, we might do:

$$21 \times 10^3$$

This might seem challenging but remember, now we know that 10^3 is really the same as 1,000. So, we could rewrite the problem like this:

$$21 \times 1000$$

This problem is exactly the same but we know exactly how to do this one already! Remember, take off the zeros, multiply and put them back?

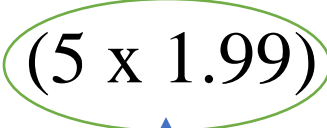
Variables

Talking about variables is the beginnings of algebra and I know that might seem scary to some but you use them all the time and don't even know it. **A variable is just a number we don't know yet!** For example. Let's say you write down all the stuff you want from store and its cost. Maybe it looks like this:

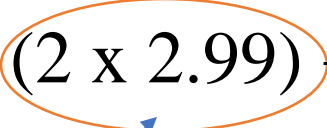
| ITEM | COST |
|-----------------------|-------------|
| Protein drink x 5 | \$1.99 each |
| Drawing paper Pad x 2 | \$2.99 |
| Bag of candy x 3 | \$2.49 each |

We'll ignore sales tax as it will just complicate things, so for now, we would simply add all the costs together like this:

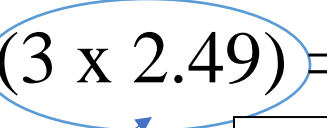
$$(5 \times 1.99) + (2 \times 2.99) + (3 \times 2.49) = C$$




Cost of 5 protein drinks



Cost of 2 drawing pads



Cost of 3 bags of candy



Total cost. We don't know it yet, so we'll call it "C"

We don't know the cost of our store order until we do the math, so it is a variable. We are calling it "C" for now. If we do the math, we'll know what "C" is. It turns out that:

$$C = \$23.40$$

Just a note here. Since "x" is often used as a variable, we will no longer use it and a multiplication symbol. So instead of:


$$3 \times 4$$

We're going write:


$$3 \cdot 4$$

Now, back to variables. Sometimes we have a relationship and we do know what the variable is, but it changes depending on the circumstances. For instance, let's say at your job you make \$30 a day plus \$10 per sale. We can write how much your pay will be like this:

$$30 + 10s$$



\$30 per day



\$10 times the number of sales you have

You'll notice that this is your pay EVERY DAY but the number of sales you have each day *varies*. ***It is a variable.*** Now, let's say one day you make 20 sales. Your pay would be:

$$30 + 10(20) = \$230$$

On another day, you may only make 5 sales in which your pay would be:

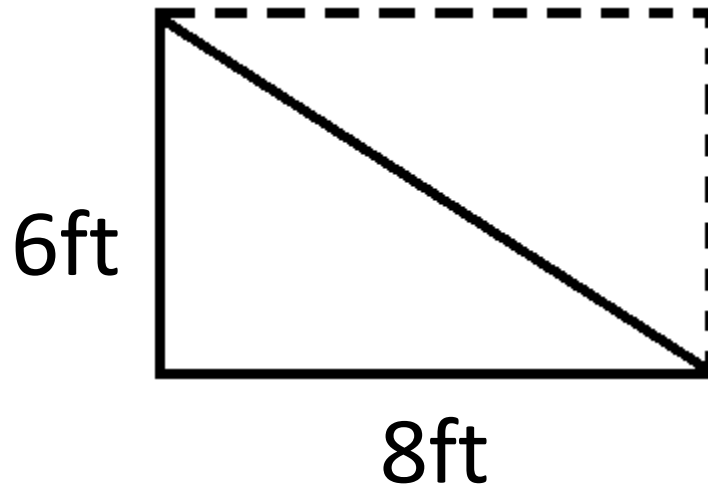
$$30 + 10(5) = \$80$$

Triangles

Remember when we talked about the area a perimeter of a rectangle? We know the lengths of the sides but not the perimeter or area. That's because they are variables until we do the math.

Remember, we talked about, the perimeter of ANY shape is simply the distance around. Easy enough.

But if you realize that a triangle is essentially $\frac{1}{2}$ a rectangle, it makes finding the area really easy. Take a look at this picture:



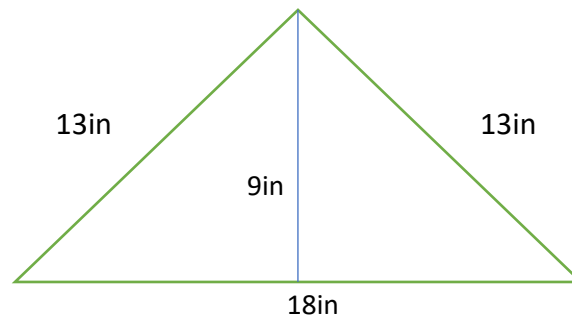
We already know that Area = length \cdot width of a rectangle. But as you can see from the picture above, there are two triangles of equal size that make this rectangle. So, the area of just one of them would be half the entire area. Thus, the area of a triangle is $\frac{1}{2}$ of the area of a respective rectangle. However, when we are talking about triangles, instead of referring to "length" and "width", we use "base" and "height." Thus:

$$A = \frac{1}{2} \cdot b \cdot h$$

Or

$$A = \frac{b \cdot h}{2}$$

Both of these are the same, so it doesn't matter which way you want to think of it. Now, let's try one. Let's say we have this triangle and we want to find the area and perimeter:

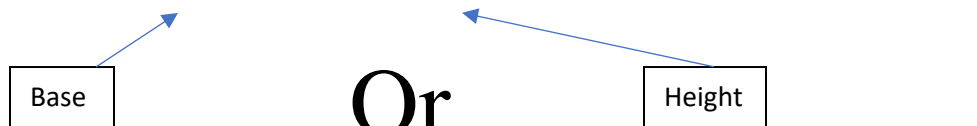


Let's find the perimeter first because that is easiest. Remember, perimeter is all the way around. So, that is just:

$$13\text{in} + 13\text{in} + 18\text{in} = 44\text{in}$$

We know how to calculate the area of a triangle and this one happens to have a base of 18in and a height of 9in. Thus, the area is:

$$\frac{1}{2} \cdot 18\text{in} \cdot 9\text{in} = 81\text{in}^2$$



Or

$$\frac{18\text{in} \cdot 9\text{in}}{2} = 81\text{in}^2$$