## SuperDuperMathTutor.com

## Square roots

You are probably aware that the reverse of addition is subtraction and the reverse of multiplication is division. It turns out that the reverse of exponents are roots. So, let's revisit exponents for a minute but just exponents of 2 . Here is 5 to the $2^{\text {nd }}$ power:


Now, it is important to remember what this means in order to fully understand square roots. Recall that $5^{2}$ means this:

$$
5^{2}=5 \cdot 5
$$

Here are some more just to help refresh your memory:

$$
\begin{aligned}
2^{2} & =2 \cdot 2 \\
19^{2} & =19 \cdot 19 \\
1^{2} & =1 \cdot 1 \\
7^{2} & =7 \cdot 7
\end{aligned}
$$

Now, recall that I said that a root is the reverse of an exponent. Just like addition is subtraction in reverse, let's do squaring in reverse. Since squaring a number means multiplying it by itself, square rooting means finding out what squared is that number. That might seem a little confusing, so let me show you some examples:


Another way of thinking of square roots is answering the question "what squared is the number under the root." For instance, let say we have this square root:

## SuperDuperMathTutor.com

$$
\sqrt{49}=?^{2}
$$

We would ask the question, "what squared is 49 ?" Of course, $7^{2}$ is 49 and so our answer is 7 . Here are some more examples to help you see the pattern here:

$$
\begin{gathered}
\sqrt{100}=10 \text { because } 10^{2}=100 \\
\sqrt{36}=6 \text { because } 6^{2}=36 \\
\sqrt{4}=2 \text { because } 2^{2}=4 \\
\sqrt{144}=12 \text { because } 1^{2}=144 \\
\sqrt{9}=3 \text { because } 3^{2}=9 \\
\sqrt{64}=8 \text { because } 8^{2}=64
\end{gathered}
$$

Is that making sense? If not, make sure you understand it because it is essential for the next bit.

## Perfect squares

You may notice that there was a pattern to the numbers I chose above. They all came out to be whole numbers, didn't they? Well, I admit I did that on purpose because it gets messy otherwise, as you'll see in a minute.

All the numbers I used are what are called perfect squares. All this means is that if I take the square root of a perfect square, you will get a whole number. Or we could also say that we can get a perfect square by squaring a number. So, $1,4,9,16,25 \ldots$.and on and on are all perfect squares because we if we take their square root we get a whole number OR we can square a whole number to get them.

## SuperDuperMathTutor.com

But there are lots more not-so-perfect squares. For example, take this one:

$$
\sqrt{47}=?
$$

$$
\begin{gathered}
\text { or } \\
?^{2}=47
\end{gathered}
$$

So, what squared is 47 ? Well, nothing, at least not a whole number. However, we do know these two facts:

$$
\sqrt{49}=7 \text { and } \sqrt{36}=6
$$

Hmmmm....So we don't know EXACTLY what $\sqrt{47}$ is but it obviously has to be somewhere between 6 and 7. But you'll notice that it is awful close to $\sqrt{49}$ which we DO know the answer to. Yet it isn't very close to $\sqrt{36}$, which is the next closest square root we know the EXACT answer to. Let's put all this visually to see what I'm talking about:


You'll notice that numbers on the top of the number line correspond EXACLTLY to the numbers on the bottom. So where would $\sqrt{47}$ go?


Well, right close to $\sqrt{49}$ as we can see mathematically and visually. If we were to estimate this number, we would say that $\sqrt{47}$ is about 6.9 . Although, what if we were asked to round it to the nearest whole number? Well, we know that it is between 6 and 7 but clearly much closer to 7 . So, we could write this:


## SuperDuperMathTutor.com

Let's talk through one more. Take this one:

$$
\sqrt{31}=\text { ? }
$$

So, what do we know? Well, we do have these two facts:

$$
\sqrt{25}=5 \text { and } \sqrt{36}=6
$$

Thus, the $\sqrt{31}$ is somewhere in between. Of course, 31 is only 5 spaces away from 36 and 6 spaces away from 25. In other words, it is just a little bit closer to $\sqrt{36}$ than $\sqrt{25}$. And so, the answer must be a little closer to 6 than 5 but not by much. So, a reasonable estimate would be a little more than $5 \frac{1}{2}$; about 5.6 or so. Here it is visually as well:


## Visual estimation

Often in life we need to know the value of something but not necessarily the EXACT value, like what we just did. So, let's start off with a pretty straight-forward chart and interpret it. So, here it is:


## SuperDuperMathTutor.com

If you look at the label on the horizontal axis, you will see that it is the number of years since 'something' started. In this case, the number of years since you opened your car repair business. The vertical axis is revenue in 1,000 's of dollars that correspond to every day in that year. In other words, how much money your business made from day 1 until 24 years later. So let me ask a few questions and see if we can answer them. Let's show the chart again, with some illustrations added to help you see the answer to these questions:


1. How much revenue did you have in the beginning of year 12 ?

If we look at year 12 it corresponds to point on the revenue curve which corresponds to aramount. It looks like about $\$ 75,000$ (per day).

## SuperDuperMathTutor.com



This one is a little bit sneaky because year 16 is not labeled but obviously it is between years 15 and 18 that are labeled. So, we'll have to eyeball it a little. If you follow the lines that I drew fromyear 16 , you can see that in year 16 you made about $\$ 50,000$ per day.

## SuperDuperMathTutor.com


3. Now for the hard one. When did you make $\$ 90,000$ per day?

The reason th/is question is more chalfenging is that instead of looking for a year and finding the amount we made, we have to find the anmount we made and what year that happened. But we are still connecting the date and arhount just like before. We just begin yith the axis for price and find $\$ 90,000$. Then we see where that corresponds to on the curve and what that point corresponds to on the year axis. It looks like you made $\$ 90,000$ in about 2013.

Now let's do another one and since I love money and I love to make money in the stock market, let's do an example with that.

We'll say that you buy stock in Amazon in 2016 and sell it in 2020. Here's what the chart of that company actually looks like up until today:


## SuperDuperMathTutor.com

So, let's estimate what you bought it at. To do this in a way that makes more sense, let me label some things for you:


Now the questions: what was the price when you bought it and what was the price when you sold it? You bought in 2016 and the big dot I put there corresponds to the price at that time as you can see if I add one more illustration:


Notice the line from the dot to the "price" axis that I just added. What price does this date correspond to? Since each tick mark is clearly $\$ 250$ and it looks like about $1 / 2$ a tick mark below $\$ 750$, that seems like about $\$ 625$ to me. Using the same logic, it looks like you sold at about \$1900. Pretty good job Mr. Moneybags!

## SuperDuperMathTutor.com

## Order of operations...again

I'm sorry if you thought PEMDAS was gone forever but it has only begun. Now we know how to deal with square roots, so where do they belong in the order of operations? I'm going to diverge from the book here and change PEMDAS to GEMDAS to better suit our needs. Now we can remember it with

## Go Excuse My Dear Aunt Sally

But now it means this:

## Groups Exponents Multiplication Division Addition Subtraction

Now the reason we are doing this is because you'll notice that 'square root' doesn't show up in PEMDAS. However, it is a 'group,' which means that it is done first, along with parenthesis. So, say we have this:

$$
4 \cdot \sqrt{49}+3^{2}
$$

What do we do first? GEMDAS tells us to do groups, which include parenthesis and square roots. We don't have parenthesis but we do have a square root, so let's do that. Now have this:

Now is exponents and we have one, so let's do that:

$$
4 \cdot 7+3^{2}
$$

$$
4 \cdot 7+9
$$

Of course, now is multiplication (there is no division):

$$
28+9
$$

This needs no explanation as we only have addition left:

$$
37
$$

Easy enough, right? Let's try one that is a little harder. Let's use a real equation that tells you how fast a car was going based on the length of its skid marks as compared to a 'test car.' It looks like this:


This looks scary I know, but we've got this. Look at the labels and you can see what all the letters mean. Now, if the test car was traveling at 50 mph , left skid marks that were 70 ft long and the target car left skid marks that are 30ft long, how fast was the target car goins?

Let's consider, that the problem just gave us v, D and d. So, let's put them in and find what V is but the trick here is that we have to be careful of the order of operations. And we will need a calculator because the numbers get a little big. Here's what we have:


When we consider GEMDAS here, groups come frst. We do have one group, a giant square root. Now when we look under the square root, w\& see that there is an exponent, multiplication and division. We do exponents before multiplication and division, so let's do that:

$$
\mathrm{V}=\sqrt{\frac{2500 \cdot 30}{70}}
$$

Now there is only multiplication and division left under he square root sign. They are equally important but you can probably see that doing multiplication first would be easiest. Let's do it:


Now division (here's where you definitely need your calculator:

## SuperDuperMathTutor.com

$$
V=\sqrt{1071.43}
$$

Now that we have done everything in the square root, we can finally take it:

$$
\mathrm{V}=32.73 \mathrm{mph}
$$

This is how fast the target car was traveling when it slammed on its breaks. You may notice as well that if you do $32.73^{2}$ you'll get 1071. It won't be exact because we rounded along the way but it will be very, very close.

## Combining like terms

This merits an analogy because it makes this concept much more clear. When I talk about "like terms" I'm talking about things that are the SAME. Not similar, the same TYPE of thing. For example, cats are the same as cats and dogs are the same as dogs, right? But dogs are not the same as cats. So, let's say you have:

## $3 \operatorname{dog} s+4 \mathrm{cats}$

Then you get married and move in with your wife and she has:

## $2 d o g s+5 c a t s$

Later someone asks you "what pets do you have at home?" What is your response? I bet you'd say this:

## "We have 5dogs and 9cats"

I was right, wasn't I? You may not realize it but you just combined like terms. Oftentimes in math we are really doing things that you already understand, but because it is "math," you get scared and confused. Let me explain what you just did.

You recognized that your dogs and her dogs are 'like terms,' meaning they are the same thing. Likewise, with the cats. Together you have this:

$$
3 \operatorname{dog} s+4 \mathrm{cats}+2 \operatorname{dog} s+5 \mathrm{cats}
$$

But this is not what you said. You simplified it by combining the 'like terms' and said this:

## SuperDuperMathTutor.com

## 5dogs +9 cats

We could also simplify it even further for our mathematical purposes like this:

## $5 d+9 c$

In math we often use letters to represent things and if the letters are the same, they are 'like' each other. Not similar, the same! For example, let's simplify this:

$$
9 x-5 y+4 x
$$

You'll notice that the $9 x$ and $4 x$ are like terms but the $-5 y$ has no like terms. So, we can combine the $9 x$ and $4 x$, but not the $-5 y$. So, I could rewrite my problem like this:

Now, recall that I can add in any order I want (associative property)? Let's rewrite it to put the like terms together:

$$
9 x+4 x+(-5 y)
$$

It's easy to see now that we can combine the $9 x$ and $4 x$ but again, the $-5 y$ is a loner:

$$
13 x+(-5 y)
$$

## Expressions vs. Equations

I find that students often confuse expressions and equations. As you already know from life, an expression is just a statement. For example:

## In words: "I have 9 bananas." or In Math: 9b

On the other hand, an equation is one expression equal to another and thus ALWAYS has an equal sign. Here are equations:

## In words: "I have 9 bananas worth $\$ 8$." In Math: $9 \mathrm{~b}=8$

## SuperDuperMathTutor.com

Now, let me give you some examples in words and in math:

## In words: "The sum of two numbers"

You may notice that "sum" refers to addition. You may also notice that there are two numbers but it doesn't tell us what they are. Remember a variable is a number that we don't know yet? That's right, we'll just use a variable. We could call the numbers "Bob" and "Susy" but to make life simpler, let's just call them $x$ and $y$. So,

$$
\text { In Math: } \mathrm{X}+\mathrm{Y} \longleftarrow \begin{aligned}
& \text { The sum of } \\
& 2 \text { numbers }
\end{aligned}
$$

How about:
In words: "The price of the dress is $\$ 30$ less than the old price"
Again, it doesn't tell us what the old price is, so we have to call it something. How about " t "? Also, "less" refers to subtraction. So, we would take the old price (which we are calling " t ") and subtract 30 :

$$
\text { In Math: } \mathrm{t}-30
$$

Now, if this was instead an equation we could have something like this:
In words: "The price of the dress is $\$ 30$ less than the old price which is now $\$ 45$ "

Like before, the first expression hasn't changed. But the old price minus $\$ 30$ is now equal to something, \$45. So, we would have this:

$$
\text { In Math: } \mathrm{t}-30=45
$$

You'll notice now that it is in equation form, we can see that the old price must have been $\$ 75$.

## SuperDuperMathTutor.com

Now, let me give you several examples and you might want to try to figure them out yourself before you look at the answer. I'll use an x for all variables just to be consistent, but remember, any letter can be used as a variable; it makes no difference. I'll do each one forwards and backwards, just so you can see the pattern. And again, we are just translating the WORDS into MATH:


## 2. Three minus twice a number <br> $3-2 x$

## 3. Triple the cost

4. Twelve divided by a number $12 \div x$
5. 



## SuperDuperMathTutor.com

Now we'll use the same expressions but equal to another expression. Or in other words, we'll have equations:

## 1. Three less than twice a number is equal to 86 <br> $$
2 x-3=46
$$

2. Three minus twice a number results in 3 times the number

$$
3-2 x=3 x
$$

$\qquad$

## 3. Triple the cost is the same as 66 $3 x=66$

4. Twelve divided by a number gives 24 less than the number

$$
12 \div x=x-24
$$

## 5. A number divided by 12 is the same as 4 $x \div 12=4$

Now you speak "MATH." But funny thing is that really already knew how, didn't you? From here on it's just practice! So, let's try a couple problems

