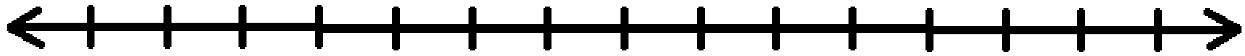
Signed numbers

All we mean by signed numbers is that all ***integers*** can be positive or negative. The integers are just whole numbers, both positive and negative. Oftentimes a number line is useful to see and use the integers. It looks like this:

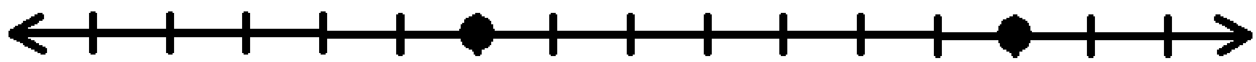


-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

The arrows at the ends just mean that it goes on forever. Also, you'll notice that as you move toward the right, the numbers get bigger. Likewise, it you move toward the left, they get smaller. And in fact, you can compare the size of 2 numbers just by plotting them on a number line and seeing which is farthest to the right. For instance, if we wanted to compare these two numbers:

$$5 \text{ \_\_\_\_ } -2$$

If we plot both of these, we'd get this:

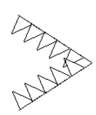


-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

You may already realize that 5 is bigger than -2 but even if you didn't, you could tell by the fact that the 5 is farther to the right on the number line. So, we would say that 5 is "greater" than -2. So, our answer will look like this:

$$5 > -2$$

When using a greater than or less than sign, always remember that the alligator "eats" the bigger number:

5  -2

Opposite of an integer

Of course, you love this class and math, so you might make a statement like:

"This is the best class ever!"

The ***opposite*** of this is:

"This is NOT the best class ever!"

It is pretty much the same in math but in math we have numbers. If we assigned a 5 to your first statement, it would only make sense that the "opposite" of it , or the second statement, would be -5.

Thus, if we want to find the opposite of an integer, we just make it negative. If it is already negative, it becomes positive. Here are example:

The ***opposite*** of 3 is -3

The ***opposite*** of -2 is 2

The ***opposite*** of 9 is -9

The ***opposite*** of -5 is 5

Negative negative

Comparing numbers is not that tough when just dealing with positive and negative numbers, but negative negatives can be a little tricky.

Every integer has an ***opposite***, meaning a negative version of itself and you can see that from the number line. The opposite of 3 is -3 and the opposite -6 is 6. Pretty easy, right? But what about when we have a negative negative. Here's an example:

$$-(-12)$$

This is a little trickier, especially if we want to find the opposite of it. But we don't even know what it means! So, let me give you a quick analogy to explain and we'll go deeper on this concept shortly. Pay close attention here:

If you have $3 and I 'take away' that $3, you have no money now. On the other hand, if you have -$3 because you lost at a card game, and I take away your debt, that would be the same, mathematically, as giving you $3, wouldn't it? Note that I 'took away' a negative, making it the same as a positive. So, if we have –(-12), it is actually the exact same as 12:

$$-(-12) = 12$$

Also, since –(-12) actually equals 12, then:

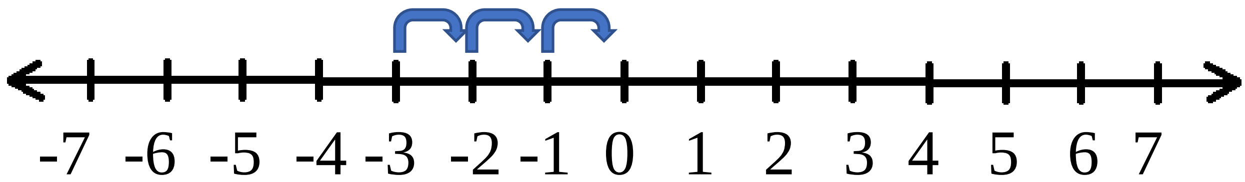
The opposite of -(-12) = -12

Absolute value

When we want to find the absolute value of something, we are simply looking for how far from zero it is. We use brackets like this to say 'what is the absolute value of -3:'

$$|-3|$$

So, what we want to determine here is how far -3 is from zero. Well, it is 3 spaces, right?:



Thus:

$$|-3| = 3$$

What about this then:

$$-|3|$$

First of all, if we take the absolute value of 3, it is just 3 because it is 3 spaces away from zero. But the negative doesn't go anywhere. In other words, whatever we get for the absolute value of 3, we make it negative. Thus:

$$-|3| = -3$$

Add and subtract integers

In the real world we deal with positive and negative numbers all the time. Positive numbers are usually used to indicate an increase of some kind like revenue or profit. Negative numbers indicate a decrease such as a loss or money that is owed. Thus, it is essential to understand how we put these numbers together.

Add two negative numbers

Let's say that your business had a loss of \$5,000 last year and \$6,000 the year before. We could say that the total would be:

$$\overbrace{-\$5{,}000}^{\boxed{\text{Loss for year 1}}} + \overbrace{-\$6{,}000}^{\boxed{\text{Loss for year 2}}} = \overbrace{-\$11{,}000}^{\boxed{\text{Total loss for both years}}}$$

You may notice that adding two negatives is the same as adding two positives, but with a negative sign. ***Adding two negative numbers will always result in a negative number***.

Add a positive and negative number

It is important to understand that ***adding a negative number is the same as subtracting a positive number***. For example, let's say that you made \$35,000 in revenue last month with a cost of \$12,000 and you want to figure out how much profit you made. The first way you could think of this is probably the most intuitive; revenue minus costs, like this:

$$\overset{\text{revenue}}{\$35{,}000} - \overset{\text{costs}}{\$12{,}000}$$

But you could also think of it as your revenue plus your costs, which are negative, like this:

$$\overset{\text{revenue}}{\$35{,}000} + (\overset{\text{costs}}{-\$12{,}000})$$

Both of these give us the same answer (\$23,000) because mathematically, they are the same.

Change a subtraction problem to addition

We could even think of it as our costs (which are negative) plus our revenue because, remember, we can add in any order we want and this includes adding negative numbers too:

$$\overset{\text{revenue}}{-\$12{,}000} + \overset{\text{costs}}{\$35{,}000}$$

Dealing with subtracting a negative

This one is a little harder to understand, so I will use an example. Let's say it's 20^o outside in the afternoon and by nightfall it is 5^o. How much did the temperature change? You probably had no trouble figuring out that the change is 15^0 because you did:

$$20^o - 5^o = 15^o$$

What you did here, whether you realize it or not, is take the starting temperature and subtract the final temperature. That gives us a change of 15^o. But what if it started out at 20^o and ended up at - -5^o? You probably already realize that this is a difference of 25^o but why? From 20 to zero and zero to -5 is a total of 25. But think about the math we just did. Remember before, we took the starting temperature minus the ending temperature, which is ***always*** how we determine a change in something (in this case temperature). If we do that here, it gives us this:

$$20^o - (-5^o)$$

But we already know that answer is going to be ***positive*** 25°, don't we? That has to mean that:

$$20^{o} - (-5^{o}) = 25^{o}$$

Notice this is POSITIVE

But if 20 – (-5) = 25 and 20 + 5 = 25, these two problems must be the same. This is why ***subtracting a negative is the same as simply adding***.

Here's another way to think about it. Let's say that you have \$5 and I have \$3. Together, we have:

My money

Your money

$$5 + 3 = 8$$

Our money together

The difference between our cash is:

My money

Your money

$$5 - 3 = 2$$

The difference between our money

Now, what if these were debts instead? You owe \$5 and I owe \$3. Together, we would "have":

My debt

Your debt

$$-5 + -3 = -8$$

Our "money" together

To find the difference between our cash (or lack thereof) would still be the same operation we did before. We just subtract two numbers. But remember, before we did your cash minus mine because you had more. Since we're dealing with debts now, I have more 'money' than you do so we do mine minus yours. But I have no doubt you already know the answer is 2. So, let's look at the math:

My debt

Your debt

$$-3 - (-5)$$

Like I said, the answer has to be 2 (that's the difference between -5 and -3, isn't it?), so:

$$\underbrace{-3}_{\text{My debt}} - (\overbrace{-5}^{\text{Your debt}}) = \underbrace{2}_{\text{The difference between our debt}}$$

But that gives us the same answer as:

$$-3 + 5$$

In other words, these are the same! And again, this is why subtracting a negative is the same as simply adding a positive.

Here are a few more examples to help you see:

$$3 - (-5) = 3 + 5$$

$$7 - (-9) = 7 + 9$$

$$-14 - (-5) = -14 + 5$$

$$-3 - (-5) = -3 + 5$$

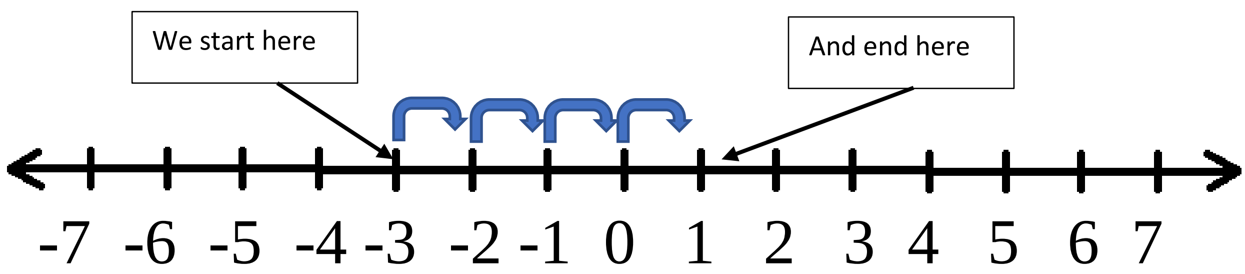
$$10 - (-5) = 10 + 5$$

Using a number line

It can also be useful to use a number line to add or subtract. When we add, we move to the right on the number line because the numbers increase in that direction. On the other hand, when we subtract, we move to the left because the numbers decrease in that direction. Let's take this problem:

$$-3 + 4$$

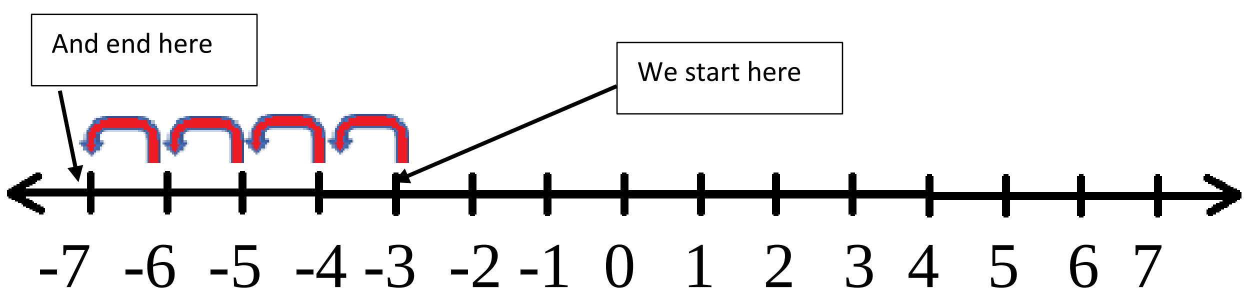
On the number line, we'll start at -3 and move 4 spaces to the right, like so:



If we change our problem to subtraction, then would have this:

$$-3 - 4$$

We still begin at -3 but we move 4 spaces to the left since subtraction makes the number smaller, like this:



So:

$$-3 - 4 = -7$$

Now what about this one:

$$-3 - (-4)$$

Hopefully you recall what we went over just a bit ago regarding negative negatives. This is actually the same as this:

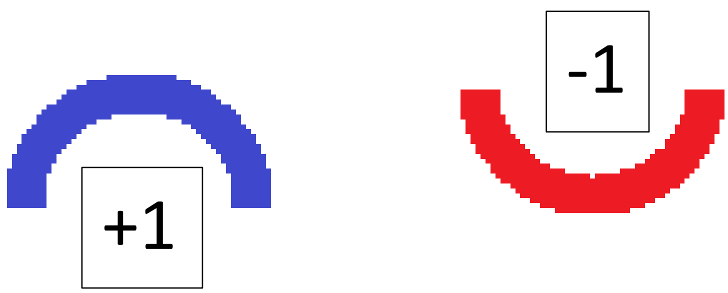
$$-3 + 4$$

This is easy because it is just the problem we started with that we've already done!

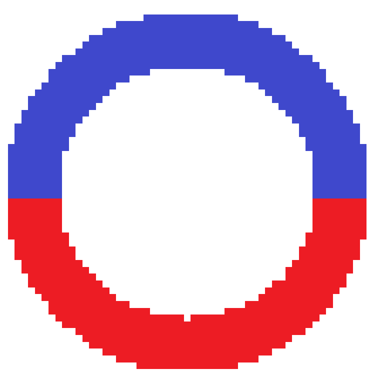
The half circles in the book

There is one concept that the book uses that I don't want you to get hung up on. It is a visual way to combine positive and negative numbers but ultimately is not any different than what I have already showed you. You might find this concept helpful and I think it is a great way to think about positive and negative numbers.

First, we represent positive 1 as a half circle that opens down and negative 1 as a half circle that opens up like this:



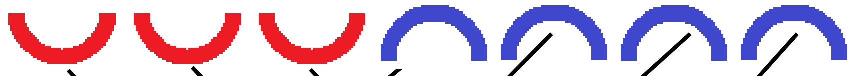
Now when we add or subtract, we can use these half circles to represent our numbers. When we have one of each, they combine to make a zero, like so:



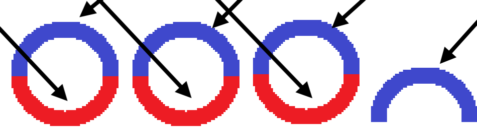
Now, let's do the same problems we just did using this method instead of a number line. And by the way, you'll probably notice that, at the end of the day, they are exactly the same. So first, we had:

$-3 + 4$

So, we need 3 negative half circles and 4 positive ones like so:



We can combine 3 of each kind to make zeros with one positive left:



As you probably expected, this gives us the same answer as we got before. Positive 1.

Let’s do the next one just to make sure you’ve got it:

$-3 – 4$

We don’t have any zeros, just 7 negatives, so:

$$-3 - 4 = -7$$

Hey, the same answer we got before. How about that! I’m starting to think that math is consistent. How about you?