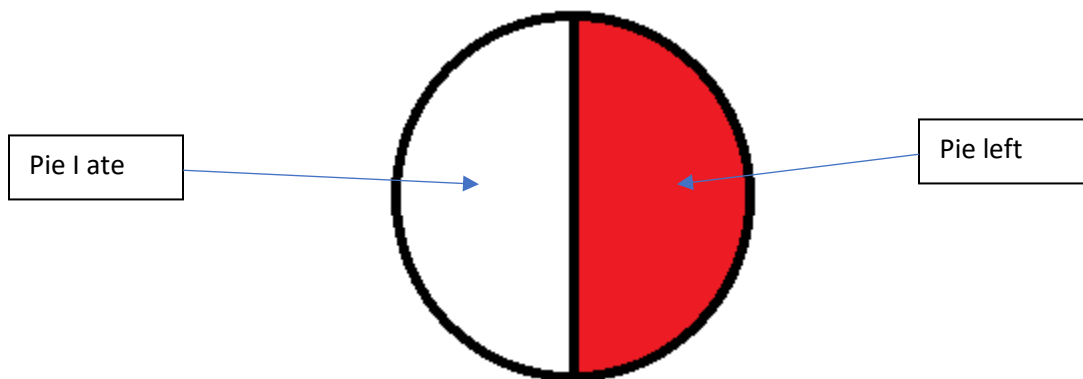


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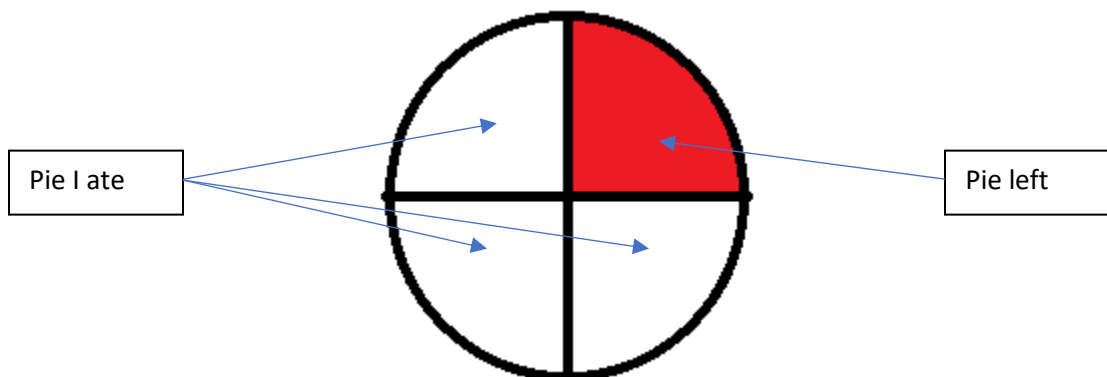
Proper Fractions

For whatever reason, fractions seem to be intimidating to students. However, you need to understand that ***fractions are just numbers***. But when we use fractions, we are talking about a ***part*** of something. Let's start off with an illustration. We'll use pies because they are easy to understand and who doesn't love pie? Let's say I eat $\frac{1}{2}$ of a pie. That looks like this:



Notice that there are 2 total pieces and I ate 1 of them. So, I ate 1 out of 2 pieces of pie. Hopefully you can see that the number of pieces I care about (1) is the number in the ***numerator***, that's the top number, and the ***denominator*** (the number on the bottom) is the total number of pieces (2).

Let's try another. This time I ate $\frac{3}{4}$ of a pie. That would look like this:



As before, do you see how this is 3 parts out of 4? There are 3 parts I care about and 4 total parts.

Both of these examples so far have been ***proper fractions***. This just means that the denominator is bigger than the numerator. These are all proper fractions because their denominator is bigger than their numerator:

$$\frac{2}{7}, \frac{1}{3}, \frac{7}{8}, \frac{3}{5}, \frac{2}{13}, \frac{3}{16}, \frac{1}{4}$$

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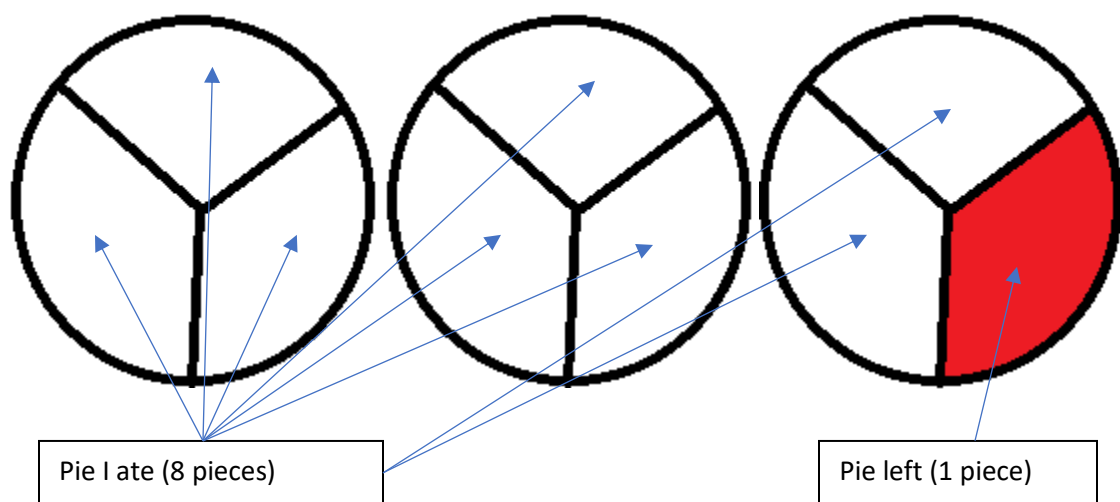
Because these are proper fractions, you'll notice that they are all a piece of something bigger. In other words, they are less than 1. 2 pieces of pie out of 7 is clearly less than 1 whole pie, isn't it?

Improper Fractions

When you are dealing with an improper fraction, you have just the opposite. Now, the numerator will be bigger than the denominator. Here are a few examples:

$$\frac{8}{3}, \frac{9}{5}, \frac{3}{2}, \frac{5}{4}, \frac{22}{8}, \frac{16}{3}, \frac{4}{3}$$

Improper fractions are always greater than 1. I know I said earlier that fractions are a part of something, which is true but sometimes that part is bigger than the whole. Think about it? If I'm really hungry and I eat $\frac{8}{3}$ of a pie that's $\frac{1}{3}$ of a pie 8 times OR eight $\frac{1}{3}$'s. Just like this:

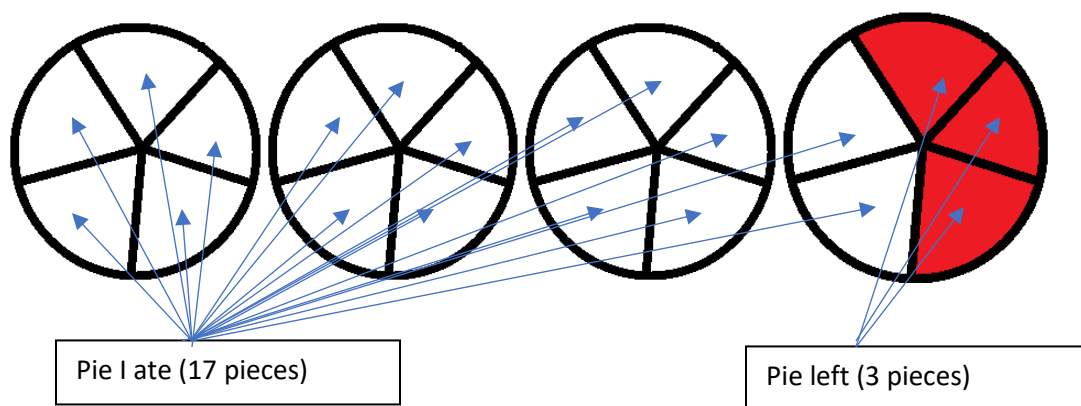


Just like last time, there are 3 pieces per pie and we care about 8 of them.

Let's look at another one with scarier numbers. But don't let it intimidate you, it is exactly the same. Pretend I'm starving and I ate this much pie:

$$\frac{17}{5}$$

So, what exactly does this mean? If we look at it, there are 17 'pieces' and the size of each is a $\frac{1}{5}$. In other words, a $\frac{1}{5}$ seventeen times. Let's see what this looks like using pies again:



Mixed numbers

The truth is you are probably not used to seeing improper fractions in real life as much as mixed numbers. After all, if I did in fact eat $\frac{17}{5}$ of a pie I certainly wouldn't tell people that. But I would tell them that I ate "3 and $\frac{2}{5}$ of a pie." You may notice that I am hinting here that:

$$\frac{17}{5} = 3\frac{2}{5}$$

I'm sure you realized that I used the same problem above and If you'll look back at it, I did indeed eat 3 whole pies and $\frac{2}{5}$ of a 4th pie. That gives a total of $3\frac{2}{5}$ or $\frac{17}{5}$. Either answer is just as correct as the other. However, the $3\frac{2}{5}$ might make more sense if you are sharing the number with someone but as you will see later, $\frac{17}{5}$ makes our lives a lot easier if we are going to do something with this number.

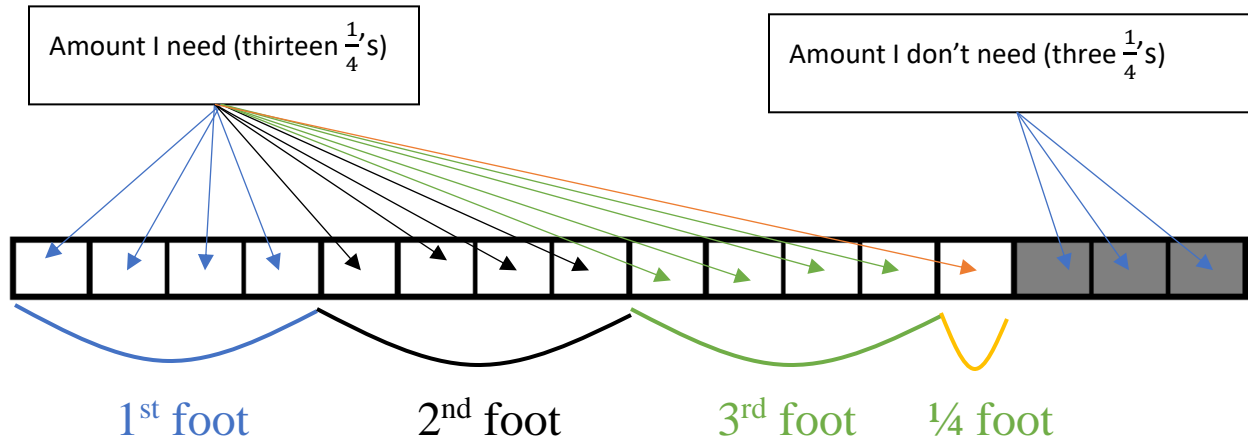
As you will see in a minute, you **ALWAYS ALWAYS ALWAYS** want to use improper fractions rather than mixed numbers when you are adding, subtracting, multiplying or dividing fractions. You can still do these operations with mixed numbers, but it is a whole lot more difficult. Here's an example:

Let's say you are building an extra room onto your mansion and your friend is helping. You measure, he cuts. You measure that a board needs to be $3\frac{1}{4}$ feet long. I know you would normally use inches for this but I want to keep the numbers small so you don't get overwhelmed, so bear with me. Now, which would you tell your friend:

$$3\frac{1}{4} \text{ feet} \quad \text{or} \quad \frac{13}{4} \text{ feet}$$

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These are EXACTLY the same but it's pretty obvious which you would say. Now, if you needed 3 boards this size, it would be much easier to multiply $\frac{13}{4}$ by 3 than multiply $3\frac{1}{4}$ by 3 as you will see later. But how did we get here? Just like the problem before, if we look at a picture we can see how these are the same.



You can see that each 1ft segment is divided into 4 parts. There are 3 whole feet plus $\frac{1}{4}$ of a foot.

You might also think of it this way. Just consider the 3 feet part for a sec, forget the $\frac{1}{4}$. If we divide just the 3ft into $\frac{1}{4}$'s, we will have twelve $\frac{1}{4}$'s, like this:



Now let's bring the $\frac{1}{4}$ back. We already have twelve $\frac{1}{4}$'s, so when we add one more onto it, we get $\frac{13}{4}$. It is important for you to understand this relationship because when you do, fractions will become simple.

Mixed numbers \longleftrightarrow improper fractions

But there's a shortcut to going from a mixed number to an improper fraction too. If you understand thus far, there is a good chance you will see WHY this shortcut works, not just how to use it. Now, let's use our familiar $3\frac{1}{4}$. Notice that if we multiply the 4 by the 3 and add 1, we

get $\frac{13}{4}$ like this:

$$\begin{array}{c}
 + \\
 \uparrow \\
 3\frac{1}{4} \rightarrow 4(3) + 1 \rightarrow \frac{13}{4} \\
 \uparrow \\
 \text{X}
 \end{array}$$

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Of course, we can go the other direction too: a mixed number to an improper fraction. We just have to think in reverse. Let's take $\frac{13}{4}$ once again. We already know the answer of course but let's think about why. We know that four $\frac{1}{4}$'s make a whole, eight $\frac{1}{4}$'s makes two wholes, and twelve $\frac{1}{4}$'s makes 3 wholes. That's twelve $\frac{1}{4}$'s total and we only have 13 of them. So, we have 3 wholes and $\frac{1}{4}$ left, which of course gives us $3\frac{1}{4}$.

Here are a few more examples:

$$2\frac{1}{4} = \frac{9}{4}$$

$$1\frac{3}{8} = \frac{11}{8}$$

$$4\frac{4}{5} = \frac{24}{5}$$

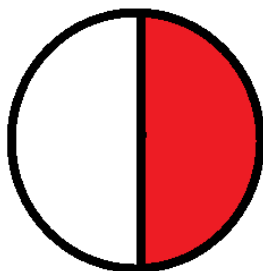
$$\frac{29}{4} = 7\frac{1}{4}$$

$$\frac{31}{5} = 6\frac{1}{5}$$

$$\frac{37}{11} = 3\frac{4}{11}$$

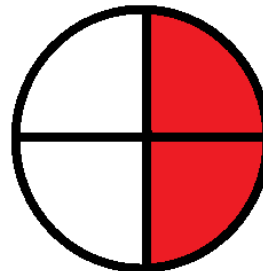
Lowest terms

With fractions, we always reduce to lowest terms, which means using the smallest whole numbers possible to express that fraction. For example, $\frac{2}{4}$ is the same as $\frac{1}{2}$ as you can see in the picture below:



1 part out of 2

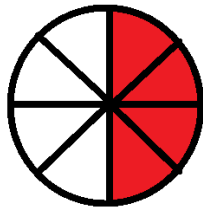
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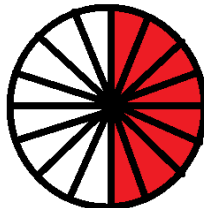
2 parts out of 4

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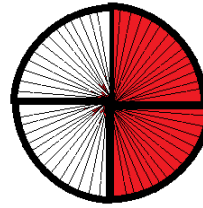
In fact, look at all these ways we can represent $\frac{1}{2}$ but with different numbers:



4 parts out of 8



8 parts out of 16



28 parts out of 56

From a mathematical perspective, if we took any of these fractions and divided the numerator AND denominator by some number, we would get $\frac{1}{2}$. Let me show you what I mean:

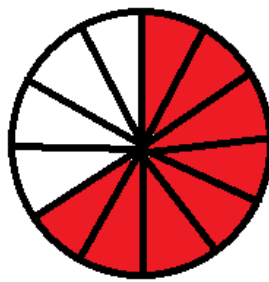
$$\frac{2}{4} \begin{array}{|c|} \hline \div 2 \\ \hline \end{array} = \frac{1}{2}$$

$$\frac{4}{8} \begin{array}{|c|} \hline \div 4 \\ \hline \end{array} = \frac{1}{2}$$

$$\frac{8}{16} \begin{array}{|c|} \hline \div 8 \\ \hline \end{array} = \frac{1}{2}$$

$$\frac{28}{56} \begin{array}{|c|} \hline \div 28 \\ \hline \end{array} = \frac{1}{2}$$

Now, we used a pretty easy number, $\frac{1}{2}$, but we do the EXACT same thing with other fractions that we can reduce. Let's pretend we have this much pie:



$$\frac{8}{12}$$

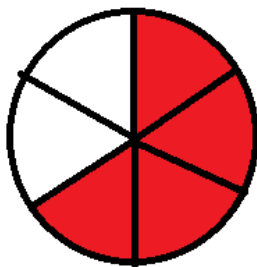
or

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First of all, is it reduced? Is there a number we can divide both the 8 and the 12 by? The first thing that comes to mind might be 2, so let's do that:

$$\frac{8}{12} \begin{array}{|c|} \hline \div 2 \\ \hline \div 2 \\ \hline \end{array} = \frac{4}{6}$$

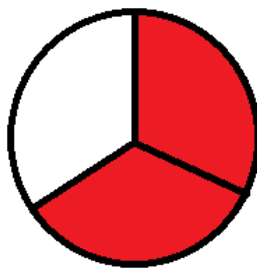
Or



Well, look at that! Now we have smaller numbers but they can still both be divided by 2. So, let's do it again:

$$\frac{4}{6} \begin{array}{|c|} \hline \div 2 \\ \hline \div 2 \\ \hline \end{array} = \frac{2}{3}$$

Or



Notice that the amount of pie left was EXACTLY the same, even though the size of the slices changed. You may have also noticed that at the beginning we could have just divided by 4 because 4 goes into 8 and 12. But in a sense we did because we divided by 2 twice, which is the same as dividing by 4. Let's do it just so I can show you that it is EXACTLY the same answer and it's OK if you have to divide multiple times rather than doing it all at once:

$$\frac{8}{12} \begin{array}{|c|} \hline \div 4 \\ \hline \div 4 \\ \hline \end{array} = \frac{2}{3}$$

Sometimes when we need to reduce larger fractions, it is difficult to see what the biggest number they can be divided by is. We call the biggest number that can be divided into 2 numbers the ***greatest common factor*** or ***GCF***. It isn't essential to find it but what often happens is that you'll end up reducing several times like we just did above. Fortunately, there are "rules" to be able to tell what a number is divisible by. They are found in your book and I won't cover them here, but we'll do an example. Let's say we have this fraction:

$$\frac{210}{234}$$

We can tell that both of these numbers are divisible by 2 because they are even (which is also one of the rules). However, if there is an even bigger number, we'll have to divide again. So, why don't we try to find if there is a bigger number. One of the rules says that if a number ends with an even number, is divisible by 2 (which I just mentioned). Another rule says that if you add the digits and the number is divisible by 3, then the whole thing is divisible by 3. Also, any number that is divisible by 2 **and** 3, is divisible by 6. We can look and see that both of these numbers are divisible by 2 because they are even. Let's see if they are divisible by 3:

$$\begin{array}{l} 210 \rightarrow 2 + 1 + 0 = 6 \\ 234 \rightarrow 2 + 3 + 4 = 9 \end{array} \quad \begin{array}{|c|} \hline \text{Divisible by 3} \\ \hline \end{array}$$

Both of these numbers are divisible by 3 as well. Since they are both divisible by 2 and 3, they are both divisible by 6. Now that we know this, it is really easy to punch the division in our calculator and reduce this monster:

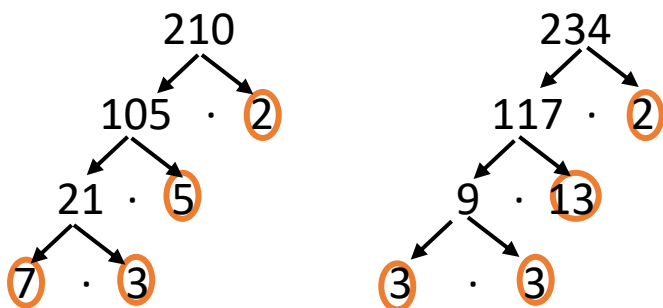
$$\frac{210}{234} \begin{array}{|c|} \hline \div 6 \\ \hline \div 6 \\ \hline \end{array} = \frac{35}{39}$$

Prime factorization

What we're really doing when we find what factors 2 numbers share, is finding what prime factors they have in common. Just a reminder, a ***prime*** number is one that cannot be divided by anything other than 1 and itself, like 3 or 11. Likewise, all other numbers are ***composite***, which means they do have factors

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in addition to 1 and themselves like 12 and 21. All numbers can be expressed as the product of their prime factors. For example, we can break down 210 and 234 from the last example like so:



In other words:

$$210 = 2 \cdot 5 \cdot 3 \cdot 7 \text{ and } 234 = 2 \cdot 13 \cdot 3 \cdot 3$$

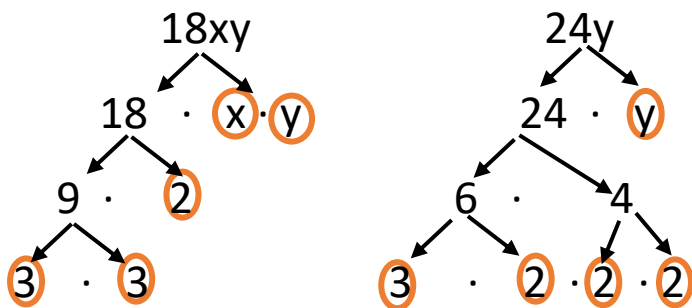
It is interesting isn't it, that the only prime factors these share are 2 and 3, whose product is 6?

This is what we determined just a minute ago; the **GCF** of these two numbers.

We can also apply this to 'scary' fractions that have letters in them (which are just numbers we don't know). For example:

$$\frac{18xy}{24y}$$

From what we already know, it is easy to see that the 18 and 24 can both be divided by 6. The xy in the numerator as well as the y in the denominator can both be divided by y because y is the **GCF**:



From this we can see that:

$$18xy = 3 \cdot 3 \cdot 2 \cdot x \cdot y \text{ and } 24y = 3 \cdot 2 \cdot 2 \cdot 2 \cdot y$$

Both of these numbers have a 2, 3 and y in common. Thus, the GCF is $6y$. So, we can divide both the top and bottom by $6y$:

$$\frac{18xy}{24y} \boxed{\begin{array}{c} \div 6y \\ \div 6y \end{array}} = \frac{3x}{4}$$

Higher terms

Basically, there are an infinite number of ways to express a fraction but only ONE way to express it in lowest terms. That's why it is so important. However, there are times as you will learn soon, that making a fraction bigger has its uses too. That's really easy because we've been taking bigger fractions and making them smaller and now we're just doing the opposite. So, let's just take $\frac{1}{2}$ again. You want to eat $\frac{1}{2}$ a pie, but it is sliced into $\frac{1}{8}$ size pieces. How many do you eat? Well, if it were cut into $\frac{1}{2}$ size slices, then you would eat 1 piece out of 2. So, if there are 4 times that many pieces (8), that means you will have to also eat 4 times as many pieces. In other words:

$$\frac{1}{2} \boxed{\begin{array}{c} \cdot 4 \\ \cdot 4 \end{array}} = \frac{4}{8}$$

This fact becomes essential in a bit.

Adding and subtracting Fractions

Fractions with the same denominator

Adding and subtracting fractions that already have the same denominator is pretty straightforward. In fact, I bet you can already do it. How about this problem to warm up:

$$3 \text{ apples} + 4 \text{ apples}$$

I'm betting you came up with 7 apples because you added 3 things to 4 of the same things. That's 7 things, in this case, apples. Now what if we had this instead:

$$\text{Three } \frac{1}{4}\text{'s} + \text{Four } \frac{1}{4}\text{'s}$$

Or



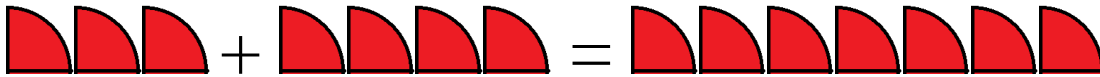
Or

$$\frac{3}{4} + \frac{4}{4}$$

These are all exactly the same and each way we think about it, we get:

$$\text{Three } \frac{1}{4}\text{'s} + \text{Four } \frac{1}{4}\text{'s} = \text{Seven } \frac{1}{4}\text{'s}$$

Or



Or

$$\frac{3}{4} + \frac{4}{4} = \frac{7}{4}$$

That was almost too easy, wasn't it? Here are a few more examples for you to take a look at in case you are still not totally getting it. And remember, an improper fraction can also be expressed as a mixed number. By the way, I personally prefer improper fractions to mixed numbers because they are more mathematically useful, but unless I ask for one way over the other, I don't care which one you give me. Anyway, here are the examples I promised IN REDUCED FORM:

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5} \text{ or } 1\frac{1}{5}$$

$$\frac{5}{4} + \frac{3}{4} = \frac{8}{4} \quad \boxed{\begin{array}{c} \div 4 \\ \hline \div 4 \end{array}} = \frac{2}{1} = 2$$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} \quad \boxed{\begin{array}{c} \div 2 \\ \hline \div 2 \end{array}} = \frac{1}{2}$$

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

$$\frac{7}{4} + \frac{8}{4} = \frac{15}{4} \text{ or } 3\frac{3}{4}$$

$$\frac{9}{2} + \frac{3}{2} - \frac{1}{2} = \frac{11}{2} \text{ or } 5\frac{1}{2}$$

Fractions with DIFFERENT denominators

Obviously when we need to add or subtract fractions, we won't always get fractions with the same denominator. For example, what if we eat $\frac{1}{2}$ and then another $\frac{1}{4}$? How much did we eat? Essentially we did this:

$$\frac{1}{2} + \frac{1}{4}$$

By now you should realize that $\frac{1}{2}$ is the same as $\frac{2}{4}$, so we could rewrite it this way:

$$\frac{2}{4} + \frac{1}{4}$$


If we do this, now the ‘pieces’ are the same size and all we have to do is add 2 of something and 1 of something, in this case, $\frac{1}{4}$ ’s, which gives us three $\frac{1}{4}$ ’s or:

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Remember back to just recently where we increased the numerator and denominator by the same number to get a ‘bigger’ fraction that it is actually the EXACT same value? All we have to do when we add or subtract fractions with different denominators is change one or both of the fractions so that the denominators are the same. The smallest denominator we can get for BOTH fractions is called the *least common denominator*. For instance, let’s take this problem:

$$\frac{1}{4} + \frac{3}{8}$$

You’ll notice that we could make the $\frac{1}{4}$ into units of $\frac{1}{8}$ ’s by multiplying by 2, like so:

$$\frac{1}{4} \begin{array}{|c|} \hline \cdot 2 \\ \hline \cdot 2 \\ \hline \end{array} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$


Not too tough I hope but let’s do at least one more:

$$\frac{5}{6} - \frac{3}{8}$$

This one is a little trickier because we are obviously going to have to change BOTH denominator. So, let’s think. What can 6 and 8 both go into? 24, 48, 72,...actually there are an infinity of things and truly it doesn’t matter which one we pick, but the smallest one always gives us the shortest problem, so let’s use 24. Now we do this math to make BOTH denominators into 24:

$$\frac{5}{6} \begin{array}{|c|} \hline \cdot 4 \\ \hline \cdot 4 \\ \hline \end{array} - \frac{3}{8} \begin{array}{|c|} \hline \cdot 3 \\ \hline \cdot 3 \\ \hline \end{array} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$$

Now what would have happened if we had used another number for the denominator, like 48? Let's try and see. After all, we **SHOULD** get the same answer, right? Now instead of using 4 and 3 as our multipliers, we're going to use 8 and 6. We get this:

$$\frac{5}{6} \begin{array}{|c|} \hline \cdot 8 \\ \hline \cdot 8 \\ \hline \end{array} - \frac{3}{8} \begin{array}{|c|} \hline \cdot 6 \\ \hline \cdot 6 \\ \hline \end{array} = \frac{40}{48} - \frac{18}{48} = \frac{22}{48}$$

=

$$\frac{22}{48} \begin{array}{|c|} \hline \div 2 \\ \hline \div 2 \\ \hline \end{array} = \frac{11}{24}$$

Notice we can reduce this!

Sure enough, same answer, but we ended up needing to reduce at the end. So, when we are adding or subtracting fractions, we find a **common denominator**, or a number that both denominators can be made into. If it is the **smallest** possible number, we call it the **least common denominator**.

Here are a couple more examples that are worked out for you:

$$\frac{8}{9} - \frac{2}{3}$$

=

$$\frac{8}{9} - \frac{2}{3} \begin{array}{|c|} \hline \cdot 3 \\ \hline \cdot 3 \\ \hline \end{array}$$

=

$$\frac{8}{9} - \frac{6}{9} = \frac{2}{9}$$

$$\frac{1}{5} + \frac{5}{6}$$

$$\frac{1}{5} \begin{array}{|c|} \hline \cdot 6 \\ \hline \cdot 6 \\ \hline \end{array} + \frac{5}{6} \begin{array}{|c|} \hline \cdot 5 \\ \hline \cdot 5 \\ \hline \end{array}$$

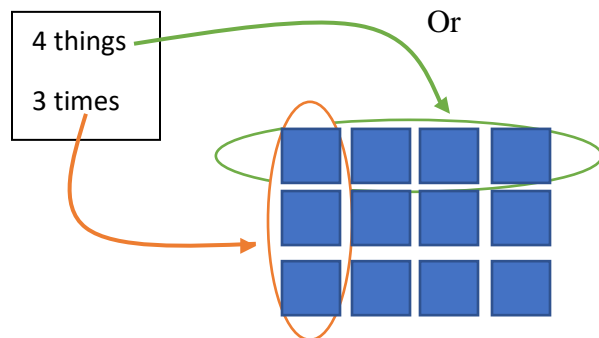
=

$$\frac{6}{30} + \frac{25}{30} = \frac{31}{30} \text{ or } 1\frac{1}{30}$$

Multiplying fractions

You might be a little disappointed to learn just how easy multiplying fractions is but I also want you to understand WHY it is. Let's revisit multiplication from last week's lesson. Remember what multiplication *means*?

$$3 \cdot 4 = 4 + 4 + 4$$



Remember this? Good. Now I want to rewrite it a different way that might make you scratch your head but it will make more sense shortly:

$$\frac{3}{1} \cdot \frac{4}{1} = \frac{4}{1} + \frac{4}{1} + \frac{4}{1} = \frac{12}{1}$$

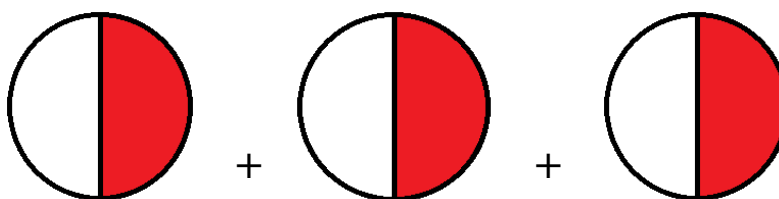
You already knew the answer would be 12 and we figured that out by adding 3 fours together, which is EXACTLY the same as multiplying 3 and 4. If we do it with ones in the denominator and by the way, that does not change the value AT ALL, you'll notice that we are essentially multiplying the numerators and the denominators:

$$\frac{3}{1} \cdot \frac{4}{1} = \frac{3 \cdot 4}{1 \cdot 1} = \frac{12}{1}$$

I want you to recall how much I have emphasize that math is EXACTLY the same so far, no matter what numbers we are using. It doesn't change. So, using EXACTLY the same logic as above, we can say that:

$$3 \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

Or



We could also write it this way since we are talking about fractions here:

$$\frac{3}{1} \cdot \frac{1}{2}$$



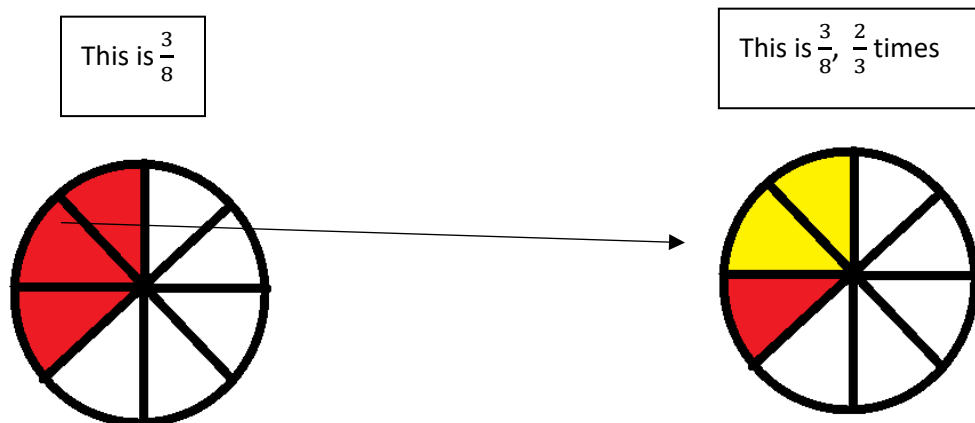
$$\frac{3 \cdot 1}{1 \cdot 2} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

In all 3 ways I wrote this, you can see that the answer is $\frac{3}{2}$ or $1\frac{1}{2}$.

Now, let's turn up the heat and look at another scarier one like this:

$$\frac{2}{3} \cdot \frac{3}{8}$$

Without confusing you, using the EXACT same logic as before, this would mean $\frac{3}{8}$ two thirds of a time. This might help:



This one may have been a little harder to follow but you'll notice that our answer is 2 out of 8 (the yellow ones) or $\frac{2}{8}$. But of course this is the same as $\frac{1}{4}$. But once again you will notice that if you multiply straight across, you get the same answer:

$$\frac{2}{3} \cdot \frac{3}{8} = \frac{6}{24} \begin{array}{|c|} \hline \div 6 \\ \hline \div 6 \\ \hline \end{array} = \frac{1}{4}$$

I showed you WHY but all we really have to do is multiply straight across. Now do you see why I said this would be disappointingly easy? And I am emphasizing again, this is UGLY with mixed numbers, so if you get them, change them into improper fractions before you try to

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multiply. It will make life so much easier. If you are a little “ify” on changing mixed numbers into improper fractions, go back and relearn that concept. Here are a few more multiplication examples to solidify this in your mind.

$$\frac{4}{5} \cdot \frac{1}{3} = \frac{4}{15}$$

$$\frac{5}{7} \cdot \frac{2}{3} = \frac{10}{21}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\frac{7}{8} \cdot \frac{7}{8} = \frac{49}{64}$$

Sometimes multiplication of fractions can be hairy because we could end up with big numbers that we have to reduce. For example, let's say we had this:

$$\frac{8}{9} \cdot \frac{3}{14} = \frac{24}{126}$$

Of course, we could reduce this, but we can also reduce it BEFORE we multiply too, which actually makes life easier. Remember, we can divide the numerator and denominator of a fraction by the same number without changing anything. First of all, the 8 (on top) and the 14 (on bottom) can both be divided by 2 like so:

$$\frac{\cancel{8}^4}{9} \cdot \frac{3}{\cancel{14}^7}$$

Likewise, the 3 on top and the 9 on bottom can also both be divided by 3:

$$\frac{\cancel{8}^4}{\cancel{9}^3} \cdot \frac{\cancel{3}^1}{\cancel{14}^7}$$

So now we have this:

$$\frac{4}{3} \cdot \frac{1}{7} = \frac{4}{21} \leftarrow \begin{array}{|l|} \hline \text{Already} \\ \text{reduced!} \\ \hline \end{array}$$

Dividing fractions

I'm sorry to disappoint you again but dividing fractions is also pretty simple when we know how to multiply. That's because when we have division we will change it to multiplication, which of course we already know how to do. Let me explain by starting off with some simple facts. I'll write them with numbers and pictures. See if you agree:

$$4 \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2} = 2$$

Or visually:

$$4 \cdot \text{half circle} = \text{half circle} + \text{half circle} + \text{half circle} + \text{half circle} = \text{two full circles}$$

Simple enough, right? We know that four, half a time is 2. Now, how about this one:

$$4 \div 2 = 2$$

or visually:

$$\text{four full circles} \div 2 = \boxed{\text{two full circles}} \boxed{\text{two full circles}} = \text{two full circles}$$

Interesting that we got the same answer, isn't it? You will notice that in the second problem that we divided, which is the reverse of multiplication. In the second problem we also divided by 2, which is the reverse of $\frac{1}{2}$ from the first problem. So, in the second problem, we used the reverse of multiplication AND the reverse of the second term. This is actually the same thing, much like walking backwards in reverse is the exact same thing as walking forwards!

This little trick is great for dividing fractions but we can use it for ANY numbers because remember, we can make any number a fraction by putting a 1 under it. Here are a few examples:

$$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \cdot \frac{3}{1} = \frac{12}{5}$$

$$\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \cdot \frac{3}{2} = \frac{15}{14}$$

$$\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2} = 1$$

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$$\frac{7}{8} \div \frac{7}{8} = \frac{7}{8} \cdot \frac{8}{7} = \frac{56}{56} = 1$$

Properties of rational numbers

Again, “rational” is a fancy word for fraction and your book uses it a lot. There are some ‘rules’ to fractions that are pretty straightforward but important to understand. Before we get to these, it is important to understand that fractions can also be expressed as division problems. For example, $\frac{1}{2}$ is the same as $1 \div 2$. Now, the rules:

- 1) Any fraction with same numerator and denominator is 1. The only exception is $\frac{0}{0}$. Here are some examples:

$$\frac{7}{7} = 1$$

$$\frac{-7}{-7} = 1$$

$$\frac{x}{x} = 1$$

$$\frac{3xy}{3xy} = 1$$

- 2) Any fraction with 1 in the denominator, is the same as just the numerator. For example:

$$\frac{7}{1} = 7$$

$$\frac{-7}{1} = -7$$

$$\frac{x}{1} = x$$

$$\frac{3xy}{1} = 3xy$$

3) Any fraction with a denominator of 0 is equal to 0. For example:

$$\frac{0}{7} = 0$$

$$\frac{0}{-7} = 0$$

$$\frac{0}{x} = 0$$

$$\frac{0}{3xy} = 0$$

4) You can't have a 0 in the denominator of a fraction. If you do, it is **undefined**. This just means it makes no sense. Recall that fractions are just division problems, so $\frac{1}{2}$ means $1 \div 2$ and also means 'how many times can 2 go into 1?' What about $\frac{1}{0}$, which also means $1 \div 0$? This can be interpreted as 'how many times can 0 go into 1?' But it can't! All the zeros in the universe still add up to 1. So:

$$\frac{7}{0} = \text{undefined}$$

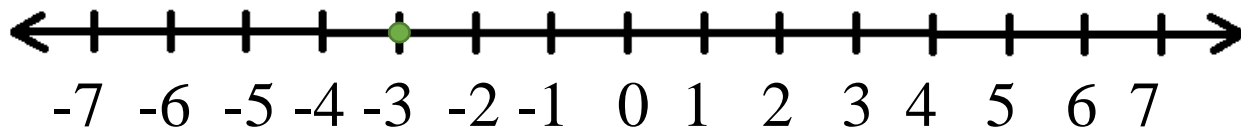
$$\frac{-7}{0} = \text{undefined}$$

$$\frac{x}{0} = \text{undefined}$$

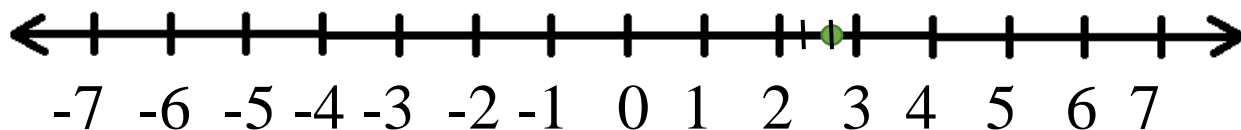
$$\frac{3xy}{0} = \text{undefined}$$

Number lines and fractions

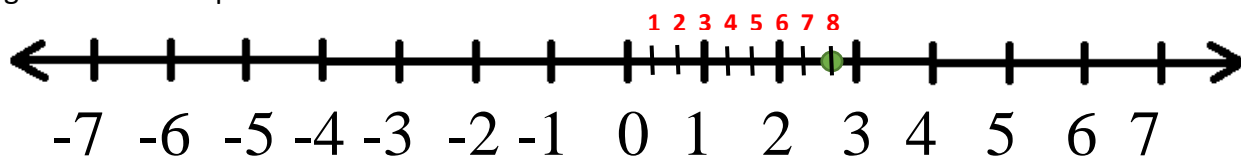
Recall from earlier sections that we sometimes used a number line. If we wanted to graph -3 on a number line, we just put a dot on the -3 spot like so:



We can do exactly the same thing with fractions. We just have to look at the “parts” in between the numbers. Let’s take $2\frac{2}{3}$ as an example (and by the way it’s almost always easier to graph mixed numbers than improper fractions.) In any case, we would go to the “2” and move $\frac{2}{3}$ farther toward the “3” like so:



Pretty simple, isn’t it. You might also notice that $2\frac{2}{3}$ is the same as $\frac{8}{3}$. If you count by $\frac{1}{3}$ ’s you will get to the same place:



Cross product

One way to tell that two fractions are equal is to use what we call **cross products**. You may know it as cross multiplication. All cross-multiplication means is that if we have two fractions equal to each other, we can multiply the numerator of one and the denominator of the other and it will always equal multiplying the other two. Let me show you what I mean using two fractions we already know are equal to each other:

$$\frac{2}{3} = \frac{4}{6}$$

Now, let’s multiply the numerator of the 1st and the denominator of the 2nd and that should be equal to the denominator of the 1st times the numerator of the 2nd like so:

$$\frac{2}{3} = \frac{4}{6}$$

$$2 \cdot 6 = 3 \cdot 4$$

Roots and exponents of fractions

In chapter 1 we talked about square roots and exponents. Here's an example of both to help you remember:

$$\sqrt{81} = 9 \text{ because } 9 \cdot 9 = 81$$

$$5^3 = 125 \text{ because } 5 \cdot 5 \cdot 5 = 125$$

When we're dealing with fractions the process is EXACTLY the same, we just have both a numerator and denominator to worry about. It also helps to know these rules, which basically say that if you square a fraction, it is the same as squaring the top and bottom separately. The same goes for square rooting:

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$$

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

So, let's say we have this problem:

$$\sqrt{\frac{81}{4}}$$

That is simply the same as:

$$\frac{\sqrt{9}}{\sqrt{4}} \downarrow \frac{3}{2}$$

What about this one:

$$\left(\frac{4}{5}\right)^2$$

That's the same as:

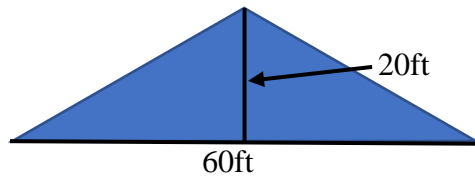
$$\frac{4^2}{5^2} \downarrow \frac{16}{25}$$

Area of a triangle

Now let's apply all of this knowledge to a real-life application: triangles. We see them all the time and we often need to know their area, or the amount of space inside them. It turns out that every triangle's area can be found with this formula:

$$\underset{\substack{\uparrow \\ \text{Area}}}{A} = \frac{1}{2} \cdot \underset{\substack{\uparrow \\ \text{Base}}}{b} \cdot \underset{\substack{\uparrow \\ \text{Height}}}{h}$$

For example, let's say you are needing to paint the area under the roof of the front of a barn. It looks like this:



The base is 60ft and the height is 20ft, so the area is:

$$A = \frac{1}{2} \cdot 60 \cdot 20$$



$$600\text{ft}^2$$