Adding rational expressions

Last week we left off with rational expressions, which, if you recall is a just a fancy term for ‘fraction.’ We added fractions but only those with numbers, no letters. However, we still have to be able to add numbers, including rational numbers that consist of numbers AND letters. But here’s the easy part; it is EXACLTY the same.

If they have the same denominator, even if it is ugly, all we do is add (or subtract). For instance, let’s take this problem:

$$\frac{4}{x} - \frac{3}{x}$$

Since the denominator is the same, we can just subtract 3 from 4 like so:

$$\frac{4-3}{x}$$

$$\frac{1}{x}$$

Notice this is EXACTLY the same thing we did before, it just looks scarier now because we have letters instead of numbers. Even if we have something crazy, like the following, we still do the same thing:

$$\frac{7}{axn} + \frac{2}{axn}$$

They have the same denominator, so all we do is add:

$$\frac{7+2}{axn}$$

$$\frac{9}{axn}$$

Operations with mixed numbers

In last week's lecture I talked about how to add, subtract, multiply and divide using fractions. But you'll notice that I never used a mixed number. But you did learn how to change a mixed number into an improper fraction. Thus, you know how to change ANY mixed number into an improper fraction and you also know how to do all the operations with improper fractions. Therefore, you already know how to do operations with mixed numbers. All you have to do is change them to improper fractions first!

Here's an example:

$$2\frac{2}{3} - 1\frac{1}{4}$$

We already know how to change these into improper fractions, so let's do it:

$$\frac{8}{3} - \frac{5}{4}$$

You'll notice that they don't have a common denominator, so let's fix that:

$$\frac{8}{3}\begin{matrix}\cdot 4\\ \cdot 4\end{matrix} - \frac{5}{4}\begin{matrix}\cdot 3\\ \cdot 3\end{matrix}$$

$$\frac{32}{12} - \frac{15}{12}$$

Now we can add them:

$$\frac{17}{12} \text{ or } 1\frac{5}{12}$$

Just to avoid confusion, your book will also show you another way to do a problem like this without ever changing the mixed numbers to improper fractions. It is completely correct but in all the math I have done over the years, I have found with little exception, that while mixed numbers make good descriptors (for example, describing how many cups of flour in a batch of cookies), they are lousy when it comes to doing math (like tripling the number of cups of flour).

But since your book does it, I will show you how to do it. In terms of your homework and final, how which 'method' you use doesn't matter. So, here's the same problem using "method 2." Here's the problem again:

$$2\frac{2}{3} - 1\frac{1}{4}$$

The first thing I'll do is line them up like you are used to when doing math by hand:

$$\begin{array}{r} 2\frac{2}{3} \\ -\ 1\frac{1}{4} \\ \hline \end{array}$$

But we still need common denominators of course, so let's change that:

$$\begin{array}{r} 2\frac{2 \cdot 4}{3 \cdot 4} \\ -\ 1\frac{1 \cdot 3}{4 \cdot 3} \\ \hline \end{array}$$

$$\begin{array}{r} 2\frac{8}{12} \\ -\ 1\frac{3}{12} \\ \hline \end{array}$$

Now we subtract in the same way we have since grade school. We line up the place values that are the same and subtract. The 2 and 1 are the same, so we do that:

$$\begin{array}{r} 2\frac{8}{12} \\ -\ 1\frac{3}{12} \\ \hline 1\phantom{\frac{3}{12}} \end{array}$$

Now we'll take care of the fraction part:

$$\begin{array}{r} 2\frac{8}{12} \\ -\ 1\frac{3}{12} \\ \hline 1\frac{5}{12} \end{array}$$

Same answer! It should be, right? But like I said, this is not my preferred way to do it. Improper fractions are always easier to deal with when you are doing math. Here's part of the reason why. Let's do this problem:

$$\begin{array}{r} 3\frac{5}{8} \\ +\ 2\frac{3}{4} \\ \hline \end{array}$$

First a common denominator:

$$
\begin{array}{r}
3\frac{5}{8} \\
+\ 2\frac{6}{8} \\
\hline
\end{array}
$$

Now we add the respective columns and get:

$$
\begin{array}{r}
3\frac{5}{8} \\
+\ 2\frac{6}{8} \\
\hline
5\frac{11}{8}
\end{array}
$$

But $\frac{11}{8}$ is the same as $1\frac{3}{8}$, so our answer is really:

$$5 + 1\frac{3}{8}$$

$$\Downarrow$$

$$6\frac{3}{8}$$

We're still ultimately doing the same thing, it just requires a little more math. Doing subtraction this way can be even worse because you can end up with a negative fraction and then you have to add part of the whole number onto the fraction to make it positive like this:

$$\begin{array}{r} 4\frac{1}{3} \\ -\ 2\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 4\frac{1}{3} \\ -\ 2\frac{2}{3} \\ \hline 2\frac{-1}{3} \end{array}$$

This is a weird answer for sure because it isn't finished. We can take a $\frac{3}{3}$ (which is equal to 1) away from the 2 and add it to the $\frac{-1}{3}$ making it $\frac{2}{3}$ like this:

$$2\frac{-1}{3}$$

$$1 + \frac{3}{3} + \frac{-1}{3}$$

$$1\frac{2}{3}$$

That's a much bigger pain in the neck than just converting to improper fractions and then doing the subtraction, isn't it?

When it comes to multiplication and division, which you learned last week, you NEVER want to use mixed numbers when you are multiplying or dividing fractions. It is certainly possible but without exception, it is much more difficult than just changing to improper fractions. We won't even cover it here.