Read and write decimals

I have emphasized a lot so far that math is consistent. It is the same ALL THE TIME as you can see in the patterns that emerge constantly. Lucky for you, as we cover decimals this week, they are just like whole numbers and fractions. They are just a different way of expressing numbers. So, let's start out by defining the place values of fractions.

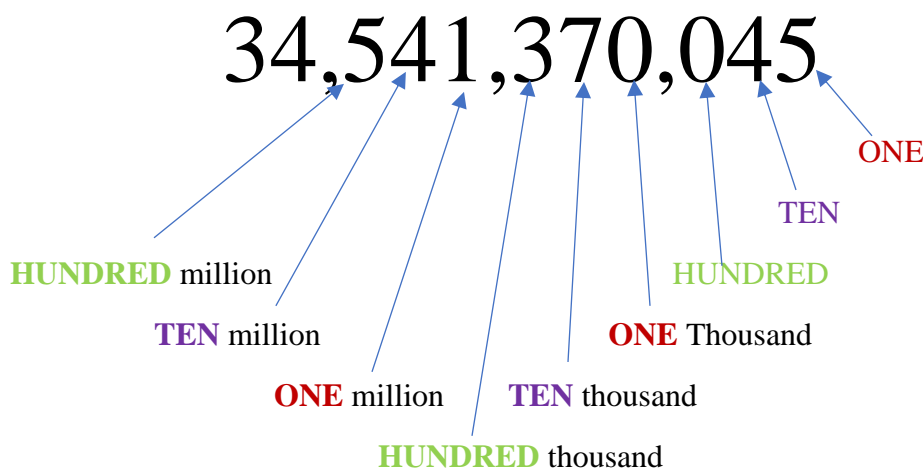At this point we are used to whole number parts such as:

$$3, 7, -5 \text{ and } -15$$

We've also done fractions, which we know to be a PART of a number and we just add them onto to any whole numbers we have or make them into improper fractions. Here are some examples:
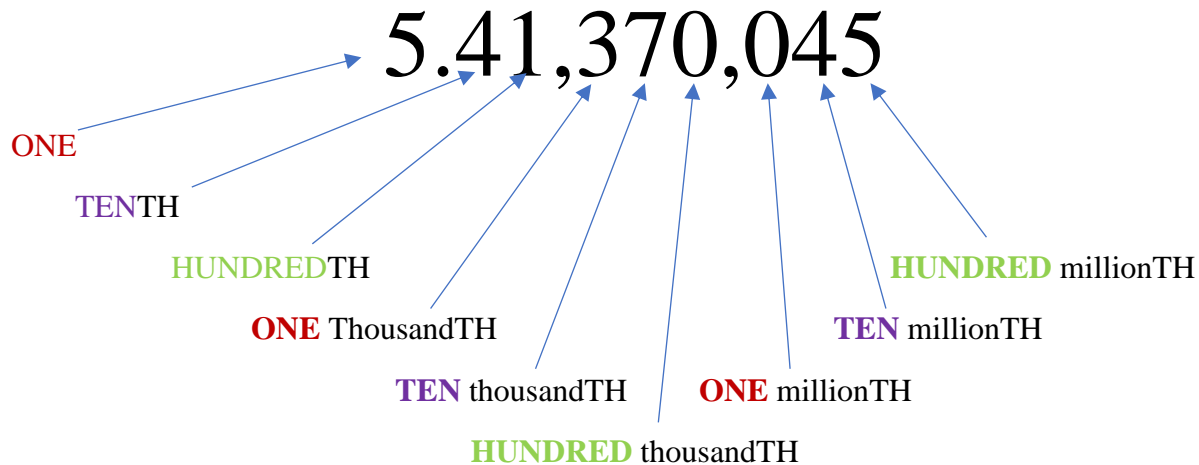
$$\frac{3}{5}, \ 4\frac{1}{2}, \ \frac{13}{7} \text{ and } -\frac{5}{3}$$

Now for the consistency. Most numbers can be expressed as a fraction AND a decimal. That is because they ARE the same thing and you will see how shortly.

But first, how to name them and write them. Here's the easy part. Decimals are written EXACLTY like whole numbers but with a "th" at the end. Here's a reminder:

$$34{,}541{,}370{,}045$$

ONE

TEN

HUNDRED million

TEN million

HUNDRED

ONE Thousand

ONE million

TEN thousand

HUNDRED thousand

This showed how we named each place value and ultimately how we named an entire number. Make sure you understand this because decimals are EXACTLY the same only in reverse of course and with a "TH" at the end of each name like this:

5.41,370,045

ONE

TENTH

HUNDREDTH

ONE ThousandTH

TEN thousandTH

HUNDRED thousandTH

ONE millionTH

TEN millionTH

HUNDRED millionTH

When we have a number and we want to name it, we do it EXACTLY the same way as we did with whole numbers by stating the value of the number but now we state the place value at the end. Let's first compare a whole number to a decimal:

3,567

We would call this number:

"Three thousand, five hundred sixty seven"

Now let's use the same number but make it a decimal like this:

0.3567

The name for this number is EXACTLY the same as before with "ten thousandths" at the end because that's the place value where it ends. It would look like this:

"Three thousand, five hundred sixty seven, ten thousandths"

Pretty easy, isn't it. EXACTLY the same as how we named numbers before, just with the name of the place value at the end. But we also must consider when there is a whole number part to it. All we do is name the whole number and the decimal part with the word "AND" in the middle. Remember I told you that I would get to when the word "AND" showed up in naming numbers? Here it is:

7654.3567

We would call this:

"Seven thousand, six hundred fifty four AND three thousand, five hundred sixty seven ten thousandths"

Again, all I did was name the whole number part and the decimal part with an "AND" between them. That's all there is to it!

Rounding decimals

When we round a decimal, the process is EXACTLY the same as rounding whole numbers like we did before. We simply determine the place value that we are rounding to and look at the number after it. If it is a 5 or higher, the number goes up and everything else becomes a zero. If not, it stays the same and everything still becomes a zero. But with a decimal, zeros at the end don't mean anything, just like zeros in front of a whole number don't mean anything. Let me show you:

0009245 is the same as 9245

0.9245000 is the same as 0.9245

But now to actually rounding. Remember this problem from week 1?:

19,283,746

We rounded it to each place value, so let's make it a decimal and do the same. I have made this to beautifully correspond to your week 1 lecture, so it would be good to get that out and compare.

Here's the number and notice that commas are not normally used in decimals:

0.19283746

First, let's round to the TENTH's place:

TENTHS place

0.19283746

Because the "1" is in the TENTH's place and the number after it (9) is 5 or larger, we round up. Also, 0.19283746 is closer to 0.20000000 than .010000000 so our answer is:

0.2

(remember, the zeros at the end are not necessary)

HUNDREDTHs place

If we round to the HUNDREDTHS place, we get:

0.19283746

Because the "9" is in the HUNDREDTHS place and the number after it (2) is less than 5, we keep it the same. Also, 0.19283746 is closer to 0.19000000 than 0.20000000, so our answer is:.

0.19

If we round to the THOUSANDTHS place, we get:

ONE THOUSANDTHS place

0.19283746

Because the "2" is in the THOUSANDTHS place and the number after it (8) is 5 or larger, we round up. Also, 19283746 is closer to 0.19300000 than 0.19200000, so our answer is:

0.193

If we round to the TEN THOUSANDTHS place, we get:

0.19283746

0.19280000
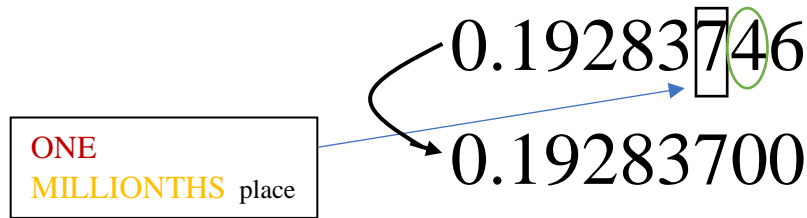
TEN THOUSANDTHS place
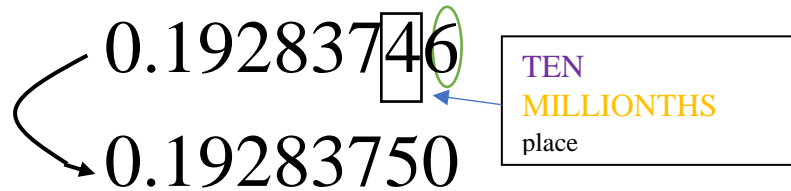
If we round to the HUNDRED THOUSANDTHS place, we get:

0.19283746

0.19284000

HUNDRED THOUSANDTHS place

If we round to the ONE MILLIONTHS place, we get:

$$0.19283\boxed{7}46$$

$$0.19283700$$

| ONE |
| MILLIONTHS place |

If we round to the TEN MILLIONTHS place, we get:

$$0.198283\boxed{74}6$$

$$0.19283750$$

| TEN |
| MILLIONTHS place |

Adding and subtracting decimals

There is a good chance you already know how to add and subtract decimals but even if you a little shaky, you'll love how easy it is. I say that because, guess what, it is EXACTLY the same as adding whole numbers.

Remember, when we add whole numbers, all we do is line up the place values and add. Of course, we also "carry" when a value exceeds the place value where we are adding. So, let's take this problem from week 1:

$$645 + 123$$

First we write it like this to make it easier:

$$\begin{array}{r} 645 \\ + 123 \\ \hline \end{array}$$

We lined up the place values and now all we do is add, like this:

$$\begin{array}{r} 645 \\ + 123 \\ \hline 768 \end{array}$$

You know this already, but we want to do a decimal problem. Let's change the numbers slightly to this:

$$6.45 + 12.3$$

Once again, let's line these numbers up by place value, just like we do with whole numbers:

$$
\begin{array}{r}
6.45 \\
+\ 12.3 \\
\hline
\end{array}
$$

You may notice this is a little strange-looking because I lined up the number according to the decimal places and there seem to be some "gaps." But remember, zeros before a whole number or after a decimal make no difference but they can be useful for us to keep everything straight. Thus, I will rewrite the problem like this:

$$
\begin{array}{r}
06.45 \\
+\ 12.30 \\
\hline
\end{array}
$$

That's a lot nicer isn't it? We haven't changed a thing; we're just making it easier to keep track of all of it. Now all we do is add exactly like we did before, just don't forget to bring the decimal down too:

$$
\begin{array}{r}
06.45 \\
+\ 12.30 \\
\hline
18.75 \\
\end{array}
$$

Simple enough, right? Let's do one with carrying but again, it is EXACTLY the same as adding whole numbers, we just have to line up the decimals:

$$
\begin{array}{r}
56.83500 \\
+\ \ 00.42589 \\
\hline
57.26089 \\
\end{array}
$$

You can see how addition is EXACTLY the same as when we had whole numbers and subtraction is too. It is EXACTLY the same, you just have to bring down the decimal.

Multiplying decimals

I love saying this yet again, and you should too because it makes your life easier: multiplying decimals is EXACLY the same as we did it before, we just have to keep track of the decimal places. For instance, given the following problem (from week 1), you'd do this:

$$^{2}34$$
$$\underline{x\ \ 7}$$
$$238$$

But in a very real sense, you just did this problem:

$$^{2}3.4$$
$$\underline{x\ \ 0.7}$$
$$2.38$$

Notice that I used the same digits but used decimals in the second problem. The same digits ended up being the answer but with a decimal. But also notice that if you count the decimal places there is 1 with the 3.4 and 1 with the 0.7. That's a total of 2 decimal places. And ironically, the answer has two decimal places, doesn't it?

All I did here was the EXACT same thing that I do with whole numbers. I just multiply and I don't even have to worry about decimal places yet. Then, when I get my answer, I apply the same number of decimal places to the answer as there were in the two numbers I multiplied. I won't get into the reason for this unless you are personally interested, I will show you in class, if we ever have it, but let's do another one for now:

Let's multiply these two numbers:

$$5.0201$$
$$\underline{x\ \ 6.22}$$

First, let's just multiply with no decimals at all:

$$50201$$
$$\underline{\times\ 622}$$
$$100402$$
$$100402$$
$$\underline{300206\qquad}$$
$$31225022$$

Now remember, we had 6 decimal places to start with:

$$5.0201$$
$$\underline{\times\ 6.22}$$

| 4 HERE + |
| 2 HERE |
| = |
| 6 TOTAL |

If we apply those to the answer we got, we have:

$$31.225022$$

Dividing decimals

It might seem repetitive by now, but guess what? Yep, division is EXACTLY the same as before. The only difference is that we are going to change the appearance of the numbers as you will see in a minute.

First, recall how we talked about fractions in week 1 and we decided that:

$$6 \div 3 = 2$$
$$60 \div 30 = 2$$
$$600 \div 300 = 2$$

We could keep repeating this if we wanted to, but you remember, right? We can use the same idea here when dividing ugly-looking numbers. For instance, let's say we have this:

$$0.006 \div 0.003$$

Or

$$0.003 \overline{)0.006}$$

Hmmm….we didn't really go over how to do something like this before, did we? So, why don't we make it into something we already know how to do. Before, as long as we moved the decimals of BOTH numbers the same number of times (by adding zeros), the answer was the same. For the SAME REASON, we can move the decimal places to the right. So, if we move both of them 3 decimals to the right, suddenly we have a really easy problem, don't we?

$$0.003 \overline{)0.006}$$

Or

$$3 \overline{)6}$$

Because we can only move the decimal places the SAME number of spaces, sometimes one number will still be a decimal. All you really care about is making the number outside a decimal. For instance:

$$0.02 \overline{)1.254}$$

$$=$$

$$2 \overline{)125.4}$$

We already know how to divide whole numbers and that's all we really have to do with this problem. Interesting, isn't it, that it is EXACTLY like division before? The only difference is now we have to make sure the decimal is accounted for. Let's do a problem just like one from week 1 but with decimals so I can illustrate:

$$0.3 \overline{)4.87}$$

This is kind of ugly of course, so let's add one decimal place to each number to get that 0.3 to be a whole number:

$$3 \overline{)48.7}$$

I encourage you to follow along with week 1 because you will see that from this point, it is EXACTLY the same, we're just going to account for the decimal when we're done:

First, we have to determine if the "3" will go into the "4" and it will, one time. But when we take one 3 from the 4, we have one left over. We still have not divided the "3" into the leftover "1" yet, so we carry it along like so:

$$
\begin{array}{r}
1 \\
3\overline{)\,48.7} \\
-3 \\
\hline
1
\end{array}
$$

> Don't forget to bring the decimal up!

Now, we still haven't divided the "3" into the "1," but 3 doesn't go into one a whole time. So, we bring down the 8 because we haven't divided into it yet either:

$$
\begin{array}{r}
1 \\
3\overline{)\,48.7} \\
-3 \\
\hline
18
\end{array}
$$

3 does go into 18, 6 times and nothing left over. That gives us this:

$$
\begin{array}{r}
16 \\
3\overline{)\,48.7} \\
-3 \\
\hline
18 \\
-\,18 \\
\hline
0
\end{array}
$$

Now we still haven't divided into the "7" yet, so we bring it down.

```
         16
    _____
 3) 48.7
    -3    |
    ____  |
    18    |
    -18   |
    ___   ↓
        07
```

3 will go into 7 two times with a remainder of 1. So now we have this:

```
         162
    _____
 3) 48.7
    -3
    ____
    18
    -18
    ___
         07
        - 6
        ___
          1
```

Here's where it diverges from what we did in week 1. We might be tempted say our answer has a remainder 1 like before. This is technically correct, but we can also express it as a decimal. You'll notice that 3 can't go into 1 and there is nothing to bring down…or is there? Actually, there's a zero there but remember, the zeros at the end of a decimal don't matter, yet we can still bring one down because even if we don't write it, it is actually there. So, let's rewrite our problem so far, with a zero:

$$
\begin{array}{r}
162 \\
3\overline{)48.70} \\
-3 \\
\hline
18 \\
-18 \\
\hline
07 \\
-6 \\
\hline
10
\end{array}
$$

As you know, 3 does go into 10, three times, so we can add a 3 to our answer with one left over again like so:

$$
\begin{array}{r}
1623 \\
3\overline{)48.70} \\
-3 \\
\hline
18 \\
-18 \\
\hline
07 \\
-6 \\
\hline
10 \\
-9 \\
\hline
1
\end{array}
$$

We can keep bringing down zeros for as long as we want or until there are no remainders. You may have already started to see what we're going to here. It will just keep repeating. Bring down a zero, we'll get a 10. Three goes into 10, three times with 1 leftover. Bring down a zero

and another 10. So now all we have to do is account for the zero, which simply means bringing the decimal place up to where it belongs like so:

$$3\overline{)48.70} = 16.23$$

So, although the actual answer is:

16.2333333333333 (forever)

We just rounded it to the hundredTHs place because we aren't really interested in writing 3's forever.

Decimals ➔ fractions

It turns out that turning a decimal into a fraction or a fraction into a decimal is pretty darn simple IF you understand what each means. I will start with a fraction and apply ONLY what you already know to convert it to a fraction. Here's the number:

5.13

All we really have to do is NAME this name and it tells us exactly what the fraction is. The name would be:

"Five AND 13 one hundredths"

If I had said 1 one hundredth, the fraction would clearly be this:

$$\frac{1}{100}$$

So, what if I had 5 and 13 one hundredths? We'll, we'd just change out the 1 for 513 and get this:

$$\frac{513}{100}$$

WOW! That was incredibly straight-forward. It's almost like the name of a decimal *IS* the name of the fraction. Let's try another:

0.0002

Since the 2 is in the 10 thousandths place, the name of it is:

"Two ten thousandths"

If we write that as a fraction it is simply 2 out of 10 thousand like this:

$$\frac{2}{10,000}$$

But wait, we're not done yet. Remember, fractions need to be in lowest terms, and you'll notice that both the numerator and denominator can be divided by 2. So, we have:

$$\frac{2}{10,000} \boxed{\begin{array}{c} \div\, 2 \\ \hline \div\, 2 \end{array}} = \frac{1}{5,000}$$

fractions ⟶ decimals

The transition from fractions to decimals is very straight-forward as well because a fraction is just a division problem and as you just learned a bit ago, division problems give us decimal answers if they don't divide evenly. For instance, let's take:

$$\frac{3}{5}$$

That is literally the same as this:

$$3\overline{)5}$$

Division is easy enough, especially on a calculator and we get:

0.60

It's hard to express how fun this is! Let's try another one:

$$4\frac{1}{8}$$

OR

$$\frac{33}{8}$$

(remember, improper fractions are always easier to deal with!)

OR

$$8\overline{)33}$$

When we do this simple division, we get this:

$$4.125$$

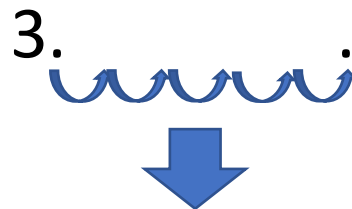## Multiplying or dividing by multiples of 10

We can already do simple multiplication as follows:

$$3 \cdot 100{,}000$$

⬇

$$300{,}000$$

Notice that we multiplied by 100,000, which has 5 zeros. Our answer also has 5 zeros, which is the same as having moved the decimal place on the 3, five spaces to the right:

$$3. \underset{\smile\smile\smile\smile\smile}{} .$$

⬇

3.00000.

Do you see where I'm going with this? Precisely the same thing happens when we multiply a decimal by a power of 10. We just move the decimal to the right by the number of zeros. For instance:

$$5.86 \cdot 1,000,000$$

6 zeros

5.86

6 decimal places

5,860,000

You probably won't be surprised to learn that division is exactly the same, we're just moving the decimal to the left instead of the right. So:

$$5.86 \div 1,000$$

3 zeros

5.86

3decimal places

.005.86

0.00586

A note about fractions and decimals

From a mhematical perspective, fractions are ALWAYS better than decimals because fractions are ALWAYS exact whereas decimals might not be. For instance, let's say we have the fraction:

$$\frac{1}{3}$$

Suppose you get this much inheritance and there are two others that each get $\frac{1}{3}$ as well. Three parts, you get one, right? But this number as a decimal is this:

0.33333333333333333333333333333333333 and it goes forever

Thus, if we want to use a decimal, we have to round it. But no matter what we round it to, it is always at least slightly less than $\frac{1}{3}$. That's especially no good when we're talking about money!

On the other hand, sometimes there are decimals that can't be made into fractions because they go forever AND they don't repeat. For example:

3.14159265358979323846264338327950288419716939937510...

and it keeps going forever, never repeating

This is a real number that is used all the time in practical math and geometry. But it never ends and so you can never find what place value it ends at because it doesn't end. Thus, you can't make it into an exact fraction. So, if we need to use it, we just round to as many decimal places as we need for whatever we're doing. Obviously if you are building a garden, you won't need very many decimal places but if you are building a nearly microscopic computer chip, a lot of decimals are probably necessary.